Economic Growth and the Environment: The Environmental Kuznets Curve and Sustainable Development in an Endogenous Growth Model

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Abstract

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In this dissertation we develop a model of human capital with differentiated physical capital, which incorporates the environmental externality of pollution, in order to investigate the interaction between economic growth and the environment in three major ways.

First, we analyze our model to provide a theoretical basis for the empirical evidence of an inverted U-shaped relationship between growth and pollution. It is shown that, in both static and dynamic stages, the timing and strictness of pollution control depend on the potential output level, and that the optimal behavior of pollution displays an inverted U-shaped pattern if the elasticity of marginal utility of consumption is greater than one.

Second, we explore the long-run growth implications in the presence of pollution for cases when pollution affects utility as a flow and a stock. Also, our model is analyzed to address the issue of sustainable development that depends not only the consumption

but also the environmental quality. We find that long-run growth and sustainable development can be achieved with the optimal control of pollution, whether pollution has its impact as a flow or stock. Sustained growth is possible as long as the social marginal product of human capital is not affected by the presence of pollution, while the cleaner type physical capital is used in production for better environmental quality. It is also found that the natural decay rate of pollution should be large enough for the existence of optimal solutions when pollution has the cumulative stock effect.

Third, we study the equilibrium growth path of a decentralized economy, and deal with the issue of implementing the social optimum by analyzing different instruments of government policy. Results show that the pollution level is unambiguously increasing in a decentralized economy, and that sustainable development cannot be achieved without government intervention. In regard to the implementation problem, it is shown that both the pollution tax and voucher schemes can implement the social optimum. We find that the effectiveness of government policy is related to the market mechanism associated with the correct price of pollution.

TABLE OF CONTENTS

LIST OF FIGURES

Acknowledgments

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Dedication

This dissertation is dedicated to my parents, without whose patient support and sacrificing love, I could have never finished what I started.

I love you!

Chapter 1

Introduction

Will pollution, as an inevitable byproduct of output, continue to increase as our economy grows, or will economic growth enable us to afford a better environmental quality? The interaction between economic growth and environmental quality has been a controversial issue addressed in both theoretical and empirical literature over the past few decades. Specifically, the question of whether or not growth can be sustained with the maintenance of environmental quality has been a worldwide concern[1]. On the one hand, some authors who explore the link between economic growth and environmental quality (e.g., Jorgenson and Wilcoxen (1990, 1992), Tahvonen and Kuuluvainen (1993)) stress the negative impact of environmental regulation on economic growth and competitiveness. They argue that since pollution abatement requires resources which otherwise could be used for productive activities, stricter environmental policy will eventually limit economic growth by raising production costs and reducing average productivities of overall inputs in the economy. On the other hand, some authors (e.g., Porter (1991), Boyd and McClelland (1999)) emphasize the importance of environmental

[1] The possible trade-off between economic growth and environmental quality has been a central issue addressed in the world-wide conferences. For example, see the Club of Rome report (1972), the World Commission on the Environment and Development (the 'Brundtland' Commission) (1987), the United Nations Conference on the Environment and Development in Rio de Janeiro (the 'Earth Summit') (1992),

preservation for sustainability of economic growth. They argue that economic activities associated with generating pollution ultimately lead to economic collapse due to catastrophic deterioration of the environmental quality if there is no environmental consideration.

Investigating the potential conflict between economic growth and environmental quality will provide important implications for the optimal strategic policies with respect to economic growth and pollution control. Developed countries that are currently concerned about the long-run effects of environmental deterioration are seeking to enforce more and more stringent environmental policies. On the contrary, less-developed countries that are concerned with fast economic growth are likely to allow less stringent environmental policies. Installing pollution abatement equipment and adopting clean technology may be very costly for less-developed countries that are currently unable to meet their desired level of investment in productive capital. Consequently, forcing them to implement stricter environmental policies will be detrimental to their economic growth because of the costs at the economy level.

Many papers in both theoretical and empirical research have addressed different aspects of the relationship between economic growth and environmental degradation. Early studies on pollution problems placed emphasis on measuring the impact of environmental regulation on the productivity and competitiveness at the industry level. At the same time, these studies motivated further research investigating the long-run effect of the optimal preservation of environmental quality on the per capita income or

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and World Bank (World Development Report) (1992).

economic growth at the economy level. A large body of work in the theoretical side of this literature has employed neoclassical growth models to examine the link between growth and the environment (e.g., Forster (1972,1973), Gifford (1973), Gruver (1976), Stephens (1976), Tahvonen and Kuuluvainen (1993), López (1994), and Selden and Song (1995)). Since these models are based on the assumption of exogenously determined growth and ignore that growth is endogenous to the economy, the typical results stress the negative relationship between the environmental quality and economic growth. Furthermore, because the change in environmental policy or the change in preference towards environmental quality does not affect the long-run growth rate in neoclassical growth models, these models are not suitable to examine whether or not growth can be sustained with the maintenance of environmental quality. On the other hand, many studies that have made contributions to the advent of new growth theory identified the key determinants of long-run growth that are endogenous to the economy, but they ignored the environmental externalities (e.g., Romer (1986, 1990), Lucas (1988), Barro (1990), and Rebelo (1991)).

If we turn our attention to the empirical side of this literature, Shafik and Bandyopadhyay (1992), Grossman and Krueger (1993, 1995), and many other papers have investigated the relationship between economic growth and pollution by using both cross-sectional and time-series data since the early 1990's. Most interestingly, substantial evidence has shown that there is an inverted U-shaped relationship between per capita income and pollution levels. Empirical evidence of such a relationship, which is often called the "environmental Kuznets curve" in the literature, suggests that economic growth might bring damage or remedy to environmental quality, depending on the current stage of economic growth. However, they do not provide a theoretical basis that explains why pollution follows such a pattern along the growth path. This is a major limitation of their studies because their interesting results are not derived from theoretical models but from the reduced-form regressions of pollution on per capita income and other related variables. In Chapter 2, we review the existing economics literature in both theoretical and empirical sides concerning the issues of economic growth and the environment.

Theoretical investigation on the interaction between economic growth and environmental degradation (pollution) is essential for at least three reasons, which provide strong motives of our research. First, it will help to clarify the difference between the conflicting views (e.g., industrialist's vs. environmentalist's) about the necessity and roles of the pollution control. Second, it will provide a theoretical basis to explain why and under what circumstances pollution follows an inverted U-shaped pattern with respect to income level, as shown in the empirical work. Third, it will provide an important implication for developing strategic policies that we should adopt with respect to economic growth and environmental regulation. If the strictness of pollution control depends on the income level, different environmental policies could be adopted for countries that are at different stages of economic growth. Thus, the main purpose of this dissertation is to study the interaction between economic growth and the environment by developing a simple theoretical model that is consistent with the empirical evidence of an inverted U-shaped relationship between growth and pollution.

In developing a theoretical model that is consistent with the empirical evidence, we try to make distinctions from the previous work in this literature. Since the analysis of pollution problems and the methods of pollution control critically depend on how we specify the nature and sources of pollution, we make a contribution to the existing literature by presenting a unique and realistic specification of the pollution generating process. Basically we treat pollution as an undesirable but inevitable byproduct of output. However, while most previous papers treat pollution as being simply proportional to final output, consumption, or inputs of production, we assume that pollution level depends on the choice of production technique in terms of cleanliness as well as the size of potential output.

In this research, we model the link between growth and the environment using the framework of endogenous growth theory for at least two reasons. First, endogenous growth models are more suitable for analyzing the interaction between growth and the environment than are neoclassical models. In particular, we are concerned with the central question of whether or not growth can be sustained with environmental maintenance. Second, we need government intervention to internalize the problem of environmental externalities, so the treatment of environmental problems could be more usefully analyzed in an endogenous growth model in which policy prescriptions affect the long-run growth paths and rates. Thus, this paper also extends the recent use of endogenous growth models to address the environmental issues.

With the development of new growth theory, many endogenous growth models have been employed to examine the link between growth and environment, but there have been very few papers addressing the role of human capital.[2] For example, Victor, Chang, and Blackburn (1994), Bovenberg and Smulders (1994, 1995), and Byrne (1997) developed models of technological changes in which they focused on examining the conditions for balanced growth path. Stokey (1998) used an *AK* model but she did not make important distinctions between the source of economic growth and that of pollution, so the result indicates that an increase in pollution is directly linked to economic growth and that optimal pollution control necessarily limits growth. On the other hand, Elbasha and Roe (1995, 1996) studied three different endogenous growth models including a human capital model to examine the link between growth and environment, but they focused only on analyzing the long-run growth of consumption without considering the welfare effect of environmental degradation.[3]

In this paper, we emphasize the role of human capital as an important source of growth in the presence of pollution control. Because the process of human capital accumulation can be interpreted as investment in education, training, or advance in knowledge, it is clear that the process of producing human capital is relatively less pollution-intensive than that of producing physical capital. We assume that human capital accumulation does not generate pollution. On the other hand, the production of physical capital generates pollution, but physical capital is differentiated in terms of

[2] Because endogenous growth models have only recently begun to be employed in examining the link between growth and environment, the human capital model has not been yet studied in this literature except Elbasha and Roe (1995). They used a human capital model and other endogenous growth models to study the long-run effect of environmental externalities on growth rates. Because they assume that pollution is simply proportional to output, economic growth leads to eventual degradation of environmental quality even in the optimal solutions of their models.

[3] See Chapter 2 for more detailed contents of previous studies that used endogenous growth models to examine the link between growth and the environment.

pollution intensities and costs so that pollution can be controlled by the choice of differentiated physical capital used in the final output production. Thus, our model has a unique and different feature from previous models in specifying the channels of interaction between growth and pollution.

In this context, we contribute to the existing literature by introducing environmental externalities into an endogenous growth model of human capital with differentiated physical capital. This is completely different from previous studies modeling the link between growth and environment, but is consistent with empirically confirmed evidence. Chapter 3 describes the basic model that will be used for investigating the interaction between economic growth and the environment throughout this dissertation. Based on our analytical framework of endogenous growth theory and its extension, we deal with different issues of concern on growth and the environment. Thus, our objectives for investigating the interaction between economic growth and the environment are not only to provide a theoretical basis for the empirical evidence, but also to explore the long-run growth implications in the presence of environmental externalities.

In Chapter 4, we present a simple theoretical model that is consistent with empirically confirmed evidence of an inverted U-shaped relationship between per capita income and pollution levels in both static and dynamic stages. By analyzing a theoretical model that incorporates environmental externalities, our goal is to provide a theoretical basis to explain why pollution follows such an inverted-U pattern in both static and dynamic settings.

If one of the important objectives for the study of economic growth is to explore its implication on welfare, then there is a strong motivation for investigating the interaction between growth and the environment. The environment affects welfare in both direct and indirect ways. If the environmental cost of growth is higher than the benefit of higher consumption, then the optimal path for the economy might be one of no economic growth. However, if the economy's increased production capacity due to continued growth could be a remedy for the environmental degradation as well as allowing higher consumption, then sustainable growth might be both feasible and optimal for the economy. Thus, Chapter 5 focuses on investigating the long-run growth implications in the presence of environmental problems. Specifically, by focusing on the long-run growth path, we examine whether unbounded growth can be sustained with the maintenance of environmental quality.

Since we are concerned about the long-run effect of environmental externalities on welfare, we incorporate the issue of sustainable development into our model. The World Commission on the Environment and Development (1987) (the 'Brundtland' Commission) defined the term "sustainable development" as development that meets the needs of the present generation without compromising the ability of future generations to meet their needs. We reinterpret the above definition of sustainable development in general as development that takes into account the welfare of future generations as well as that of present generation, which depend not only on the consumption of produced goods but also on the environmental quality as a public consumption good. Although there is no working definition of sustainable development to use in an analytical framework, we follow Byrne (1997) and Aghion and Howitt (1998) in the sense that the long-run growth of utility could be an appropriate measure of sustainable development. In this context, we make a contribution to the literature of economic growth by applying the concept of sustainable development to the model of economic growth.

In order to discuss the economic effects of various pollutants, we make a distinction between flow and stock pollutants. So, both cases are studied in Chapter 5 for the extensive analysis of the long-run behaviors of the economy in the presence of pollution.

Many recent studies analyzing the problem of environmental regulation in a decentralized economy have focused only on the trade-off relationship between growth and environmental quality (e.g., Tahvonen and Kuuluvainen (1993), Elbasha and Roe (1995, 1996), and Grimaud (1999)). They do not take into account the effect of pollution growth on the social welfare and sustainable development. By contrast, Aghion and Howitt (1998) investigate the possibility of sustainable development with optimal control of pollution in an aggregate Schumpeterian model, and they explore some conditions under which the optimal sustainable growth is possible. However, they do not deal with "the critical questions of what policies might implement the optimal sustainable growth paths that have been found" (Aghion and Howitt (1998), p.165) in a decentralized economy. On the other hand, Stokey (1998) analyzes the problem of implementing the optimal path in a decentralized economy, but her analysis is restricted to a simple *AK* model in which sustainable growth is not possible because of the diminishing returns to physical capital with the optimal control of pollution.

In Chapter 6, we study the equilibrium growth paths of a decentralized economy in the presence of an environmental externality with and without government intervention. First, when there is no government intervention, we compare the equilibrium solutions with the optimal ones that have been found by solving the social planner's problem. Also, we examine the possibility of whether or not sustainable development can be achieved in a decentralized economy without government intervention. Second, we study the issue of implementing the social optimum in a decentralized economy with government intervention. Specifically, two different kinds of policy instruments—pollution tax and pollution voucher (permit)—are analyzed in detail to see how they work in a market mechanism. We examine which of these policy instruments can implement the optimal long-run growth path in a decentralized economy.

Analytical frameworks in Chapter 6 are derived from the basic model described in Chapter 3, but the economy is decentralized into the household's and firm's problems to solve for a competitive equilibrium. When we deal with the firm's problem, we treat pollution as a normal input of production in order to analyze the firm's behavior on the input mix between pollution and other conventional inputs.

Finally, we provide a brief summary of the main results and concluding remarks in Chapter 7. Also, we discuss some important issues of our concern on economic growth and the environment, which are ignored in this dissertation, for further research.

Chapter 2

Review of the Existing Literature

2.1 Neoclassical Growth Models with Pollution

Since the early 1970's, many papers in the theoretical literature of growth and environment have addressed different aspects of pollution problems in a variety of different models. For example, D'Arge (1971) analyzed the link between the savings rate and the efficiency of investment and pollution in the framework of the Harrod-Domar model. D'Arge and Kogiku (1973) presented a simple model of waste generation to analyze an optimal control problem. Forster (1977) examined the problem of pollution control in a simple dynamic general equilibrium model and compared the competitive equilibrium solutions with the efficient solutions.

However, most of early studies in this literature employ neoclassical growth models to study the interaction between growth and environmental degradation. Based on the more or less standard assumptions of neoclassical growth models, their analyses are mostly focused on the optimal control problems of pollution and the steady state solutions. For example, authors like Keeler et al. (1972) and Gruver (1976) analyze optimal growth models, although they assume different methods of pollution control depending on the nature of pollution in their models[4], to suggest practical ways to achieve the optimal solutions for an economy with pollution problems. On the other hand, Forster (1972, 1973) and Tahvonen and Kuuluvainen (1993) focus on analyzing the steady state levels of capital and consumption when pollution is explicitly introduced to the neoclassical growth models. Not only do they analyze the properties of the steady state solutions of their models, they also discuss the implications of pollution and pollution control on the economy by comparing their solutions with those of conventional growth models in which pollution is ignored. However, because the analysis of pollution problems using neoclassical growth models is based on the assumption of exogenously determined growth, the common result from these models indicates that optimal preservation of environmental quality is necessarily in the trade-off relationship with economic growth. That is, optimal control of pollution restricts the use of productive resources, lowering the steady state levels of capital and consumption. Furthermore, because these models do not identify the key sources of the long-run growth, which can be different from the source of pollution generated, they are unable to explain the long-run impact of pollution control on growth and environmental quality.

2.2 Endogenous Growth Models with Pollution

[4] For example, Keeler et al (1972) assume that current output is used as an expenditure to reduce pollution, while Gruver (1976) assumes that the stock of capital is divided into directive productive and pollution control capital.

With the development of new growth theory in the 1990's, there have been new approaches to incorporate environmental concerns into the models of endogenous economic growth. It is a new contribution to the literature of growth and environment to integrate the new (endogenous) growth theory with environmental economics. However, most of the studies using endogenous growth models focus only on analyzing the long-run impacts of pollution on growth rates, and on examining the conditions for balanced growth. For example, the main objective of Elbasha and Roe's (1995, 1996) analysis is to investigate the implications of environmental externalities on the long-run growth rates in different classes of endogenous growth models. Because they are not concerned with the long-run consequences of growth on the level of pollution but assume that pollution is simply proportional to output, environmental quality deteriorates indefinitely with economic growth in their models. Bovenberg and Smulders (1994, 1995) examine the conditions for balanced growth in a two-sector endogenous growth model in which the natural environment is a renewable resource and pollution is defined as the extractive use of the environment. Since they focus on analyzing the long-run impact of a more ambitious environmental policy on growth, their analysis is closer to the literature on policy reform than to that on optimal policy. On the other hand, Victor, Chang and Blackburn (1994) and Byrne (1997)[5] have developed endogenous growth models in which growth depends on R & D, and on technology accumulation, respectively. Although they are concerned with the growth rate of pollution as well as the long-run

[5] The model of Byrne (1997) is closer to a static one in the sense that all output is consumed and that labor and physical capital are fixed over time. The growth rate of pollution stock, which is expressed in terms of labor and physical capital that are used for either productive or pollution abatement activity, should be necessarily fixed in the steady state. Hence, she ignores the link between pollution and output

impact of pollution control on economic growth, they place emphasis only on the balanced growth analysis. Dynamic behavior of pollution over time, which is shown to follow an inverted-U pattern in empirically confirmed evidence, is not theoretically examined in most of the studies that link the pollution problems with the theory of endogenous growth.[6]

2.3 Empirical Studies

On the empirical side of this literature, the research objectives are mainly focused on investigating the impacts of environmental regulation on industry competitiveness and economic growth. Empirical evidence has shown both positive and negative aspects of environmental regulation. Jorgenson and Wilcoxen (1990, 1992) have shown that pollution abatement spending crowds out investment in productive capital, reducing the industry productivity and the rate of economic growth.[7] By contrast, in the empirical studies by Barbera and McConnell (1990) and Jaffe et al. (1995), it was shown that there is little evidence to support the view that environmental regulations have had an adverse and significant effect on the U.S. manufacturing firms' competitiveness and growth. Furthermore, some empirical results shown by Boyd and McClelland (1999) provide

growth caused by an expansion of conventional inputs.

6 As an exception, Stokey (1998) developed an *AK* model in which growth is endogenous and the time path of total pollution displays an inverted-U pattern. However, sustainable growth is not feasible in her model with the presence of pollution because pollution control reduces the real rate of return on capital.

7 The similar results of the negative relationship between environmental preservation and industry competitiveness (or economic growth) have been shown in the empirical studies by Gollop and Roberts

evidence of a "win-win" potential for pollution abatement in the sense of both increasing output and reducing pollution, which supports the so-called "Porter hypothesis".[8] The limitation of these empirical studies, however, is that they ignore an important feature of environmental regulation at the economy level. They tend to concentrate only on the cost of pollution abatement and do not take into account the benefit from reducing pollution.

Since the early 1990's, Grossman and Krueger (1993, 1995) and other authors have investigated the relationship between economic growth and environmental degradation using both cross-sectional and time-series data. Most interestingly, substantial evidence indicates an inverted U-shaped relationship between per capita income and various types of pollution. This is also called "environmental Kuznets curve"[9] which implies that economic growth brings an initial phase of deterioration followed by a subsequent phase of improvement in environmental quality.[10] Although they provide some plausible factors that may cause an improvement in environmental quality in their empirical analysis, they do not provide a clear interpretation of their interesting results, nor do they explain theoretically why pollution follows such an

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(1983), Gray and Shadbegian (1995), and Boyd and McClelland (1999).

[8] In an article that addressed an importance of the U.S. environmental policy, Porter (1991) claimed that the conflict between environmental protection and economic competitiveness is a false dichotomy, and that environmental protection can benefit our economy's competitiveness. Boyd and McClelland (1999) viewed their method of empirical analysis as a test of this "Porter hypothesis".

[9] An inverted-U pattern of pollution relative to per capita income levels, which has been confirmed by the substantial empirical evidence, has been also called as environmental Kuznets curve due to its similarity to the pattern of inequality in the distribution of income with respect to a country's economic growth found by Kuznets (1955).

[10] Grossman and Krueger's (1993, 1995) analysis, using a cross-country panel of data, showed that ambient levels of both sulfur dioxide (SO_2) and suspended particulate matter (SPM) first rose with a country's per capita GDP, but later fell, with the turning point between $4000 and $5000 (in 1985 U.S. dollars). For the similar empirical evidence that shows an inverted U-shaped relationship between growth and environmental degradation as in Grossman and Krueger (1993, 1995), see World Bank Development Report (1992) Shafik and Bandyopadhyay (1992), Hettige et al (1992A, 1992B), Selden and Song (1994), Holtz-Eakin and Selden (1995), Carson an Jeon (1997), and Hilton and Levinson

inverted-U pattern. This is a major limitation of their studies because the observed relationship between growth and pollution is not derived from a theoretical model but from the reduced-form regressions of the level of pollution on per capita income and other covariates.

2.4 Theoretical Studies on the Inverted U-Shaped Relationship between Growth and Pollution

Along with the continued empirical evidence showing the inverted U-shaped relationship between per capita income and pollution, there have been attempts to provide a theoretical basis as well to explain why and under what circumstances pollution follows such a pattern.

López (1994) is one of the first authors in the theoretical literature on growth and environment who obtained the inverted U-shaped relationship between income and pollution. He analyzes one-way effects of growth on environmental degradation in a static framework of the neoclassical model. He shows that the inverted U-shaped curve of pollution with respect to income can be derived from his model with some specific assumptions on preferences[11], one of which indicates that the coefficient of relative risk

(1998).

[11] In the model of López (1994), preferences are non-homothetic and additively separable in consumption and pollution. Also, he argues that the coefficient of relative risk aversion is to be increased as income level increases while most of the papers in this literature assume CRRA (constant relative risk aversion) utility function. For example, in the model of growth and the environment by Bovenberg and

aversion is increasing in income. However, no evidence can be found in the literature of economics of uncertainty to support a strong positive correlation between the measure of relative risk aversion and income levels. One of the main characteristics in the production side of his model is that pollution is simply treated as a factor of production, as in Tahvonen and Kuuluvainen (1993) who justify the reason for pollution being treated as a normal input of production. However, without identifying the source of pollution or the pollution generating process, he assumes that firms will use pollution inputs until its marginal product is zero in the absence of pollution control. He ignores the fact that pollution, by itself, cannot be a productive input without an upper bound, although production might not be feasible without pollution. Hence, he does not take into account the important restriction on the feasible production technologies that use pollution as an input. The production technology should satisfy a certain boundary condition on pollution input in the sense that actual output cannot increase with pollution beyond some upper bound.

As an approach to capture a realistic feature of actual economies, such as the conflict of interest between generations, some authors analyze the potential conflict between economic growth and the maintenance of environmental quality by using the overlapping generations (OLG) models. John and Pecchenino (1994) study an overlapping generations model in which the young allocate their wages between saving

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Smulders (1995), some particular conditions on technology and preferences are derived for the balanced growth to be both feasible and optimal. They find that the utility function should be a time-separable CRRA utility function for the optimality of balanced growth. Furthermore, increasing relative risk aversion with respect to income in López (1994)'s model is only possible by ignoring the effect of third-order derivative of the utility function (i.e. u_{ccc}/u_{cc} should be negligible). On the contrary, there have been very few empirical evidence that supports a strong positive correlation between the measure of for future consumption and investment for the improvement of environmental quality in their old age. In their model, individuals' preferences are defined over consumption and environmental quality only in their old age because there is no consumption in the first (young) period of life and the environmental quality is exogenously given when young. The current period's consumption made by the old generation degrades the environmental quality in the next period. They show that an inverted U-shaped relationship between growth and pollution can be obtained depending on the presence of external increasing returns in production technology and the pattern of environmental maintenance. However, since they focus only on externalities from consumption to utility, environmental externalities that could arise from production, which could affect welfare, are excluded from their analysis. As they indicate, intergenerational externalities are intrinsically hard to internalize in the structure of an OLG model, the equilibrium solutions derived from their model can be dynamically inefficient and there may be over investment in environmental maintenance.

Jones and Manuelli (1995) develop an overlapping generations model in which the extent of pollution regulation is chosen by collective decision making by the younger generation and the rate of economic growth is determined through market interactions. They show that depending on the choice among different decision making mechanisms, the time path of pollution can display an inverted U-shape or other different patterns over time. Their objective is to analyze policies that are endogenously determined and that are essential for pollution to be controlled. They impose restrictions on preferences, and

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relative risk aversion and income levels.

define the voter's utility function over consumption and pollution in their model. It can be expressed in a simplified form that depends only on the tax rate to be determined, while other variables are predetermined. For example, although individuals participate in production activities in their first (young) period of life, the young generation is not affected by pollution and individuals derive disutility from pollution only in the second (old) period of life. Furthermore, in determining the pollution tax in their model of collective decision making, current consumption, saving, and resource allocation are assumed to be previously determined without considering the impact of future taxes. Although physical capital is allocated to both final output production and investment sectors in their model, they assume that the use of physical capital generates pollution only in the final output sector while the (dirtiest) basic capital is used for the production of new capital. If we assume that the use of physical capital generates pollution in the capital investment sector as well, the growth of the economy will not be feasible in their model because a pollution tax (or direct regulation on the quality) reduces the rate of return on capital. The analytic solutions that describe the dynamic time path of pollution are not explicitly given because of the complication of analysis in their model. Hence, it is difficult to find the specific channels or factors that make the time path of pollution display quite a different shape depending on the choice of institutions, even though they found that voting mechanism provide sufficient incentives to bound pollution.

Stokey (1998) presents simple theoretical models in which the choice of production technology determines the level of pollution and the amount of actual output. In both the static and dynamic models presented in her paper, consumption goods and pollution are simply treated as joint products of a single conventional input that also reflects the potential level of output. Hence the only way to control pollution is by determining how much to produce from the potential level of output. Although she develops an analytical framework that generates an inverted U-shape relationship between per capita income and pollution, sustainable growth cannot be obtained in her dynamic model. Since a single conventional factor is the only source of both growth and pollution in her model, sustained growth is not feasible because stricter regulation of pollution reduces the real rate of return on the single input. Even if the production technique exhibits constant returns to scale with respect to the input that is accumulated, growth will eventually cease in the presence of pollution. Also, she does not take into account the elasticity of substitution in production between conventional inputs and pollution in examining the relationship between per capita income and pollution. In a more general model of pollution as a factor of production, it turns out that the lower the elasticity of substitution in production, the faster pollution is increased with income.

Andreoni and Levinson (1998) develop a simple static model to provide the microeconomic theoretical foundations of the environmental Kuznets curve. They assume that the pollution-income relationship depends only on the technological link between the consumption of a desired good and the abatement of an undesirable byproduct, pollution. An inverted U-shaped relationship between income and pollution can be derived from their model if the pollution abatement technology exhibits increasing returns to scale. Despite their own assumption that consumption causes pollution one-for-one, it is somewhat contradictory to show that the pollution level is reduced with an increase in consumption over the range of relatively low consumption levels in their model. Moreover, since they do not take into account the production side of the economy at all, their model does not provide the dynamic time path of pollution and the long-run growth implications.

Most recently, López and Mitra (2000) explore the implications of the government's corruption and rent-seeking behavior for the relationship between growth and pollution. They studied both cases of cooperative and non-cooperative interaction between the government and the private firm that emits pollution. However, they do not present a theoretical model in which an inverted U-shaped relationship between growth and pollution can be derived. Their analysis on the relationship between growth and pollution is simply based on the previous model and results of López (1994).

Chapter 3

The Basic Model

A simple two-sector endogenous growth model is developed to investigate the interactions between growth and environmental degradation in both static and dynamic stages, and also to analyze the long-run growth implication in the presence of pollution.[12] There are two factors in the production side of the economy in this model—physical and human capital. The "general" physical capital can be transformed into infinitely many types of capital goods to be used in the final output production, which are differentiated in terms of pollution generating level and cost. With an assumption of zero population growth, we normalize the fixed amount of labor to one. Also, we assume that the worker is endowed with one unit of non-leisure time that is devoted to final output production and to human capital accumulation. Thus, the final output is produced using both human capital and the differentiated physical capital. Pollution is the inevitable byproduct that is generated in the process of final output production, and it has a negative impact on the instantaneous utility.

[12] In Chapter 4, we focus on investigating the theoretical backgrounds for the empirically confirmed evidence of an inverted U-shaped relationship between growth and pollution in both static and dynamic stages. An extensive discussion on the long-run growth impact of the optimal control of pollution will be given in Chapter 5.

3.1 Production

There are two sectors in the production side of the economy. The first sector produces a final output using two factors: a composite of differentiated physical capital goods and a human capital. The final output can be either consumed or invested for the purpose of accumulating general physical capital that will be used to produce differentiated physical capital goods.

We assume that there are N identical workers and each worker is endowed with one unit of non-leisure time, which is allocated between final output production by the fraction 1-u and human capital accumulation by u, where u is between 0 and 1. The level of human capital of each worker is denoted by h. Since we assume that population growth is zero, we normalize the number of workers to unity (i.e., N = 1) and deal with every variable as being measured in per capita terms. Thus, the effective labor devoted to the final output production is $(1-u)\cdot h$, and that to the human capital accumulation is $u \cdot h$.

Combined with human capital, differentiated physical capital is used to produce the final output. The production technology using a composite of differentiated physical capital goods is similar to that of Jones and Manuelli (1995), while most of the other parts of their model are quite different from ours. [13] A composite physical capital good

[13] Jones and Manuelli (1995) developed a two sector growth model with two-period-lived overlapping generations for the purpose of analyzing the public policies for pollution regulation that are endogenously determined. Our model is similar to Jones and Manuelli (1995) by assuming the existence of the physical

used as an input of final output production can be composed of infinitely many capital goods differentiated in terms of the marginal productivity and pollution generating levels, which are perfect substitutes in production. The differentiated physical capital goods are made from the existing stock of general physical capital. In order to avoid a trivial solution, it is necessary to assume some trade off between the economy's ability to produce more output and the choice to attain the cleaner environment by polluting less. In this context, we assume that physical capital goods that pollute less are more costly to produce. Thus, the firm's choice of how much to sacrifice from the "potential" output—an output that could be achieved by using the only "dirtiest" physical capital—for a cleaner environment depends on the firm's decision about which type (or types) of differentiated physical capital good(s) to use in production. Hence depending on the types of physical capital goods used in production, there is an infinite set of production technologies, which results in the different output and pollution levels.

The types of differentiated physical capital goods are indexed by $z \in [0,\infty)$, in such an order that higher indexed (i.e. the greater z) physical capital good is less polluting but more costly to produce. Let $p(z) = 1+z$ be the price of type z physical capital, $k(z)$, in terms of the amount of general physical capital needed to produce one unit of type z

capital stocks that are differentiated in terms of costs and pollution intensities. However, we analyze the the optimal pollution control problem in a model of an infinitely lived representative household to allow our altruistic behavior that is natural if we are at least concerned about the environmental quality we leave to the future generations. On the production side of the model in Jones and Manuelli (1995), final output is produced using a composite capital good and (a fixed amount of) labor. Because all output is consumed, growth of the economy is obtained through the production of new capital. While the "general" capital is split into differentiated capital goods to be used in the production of consumption goods, the "pure" investment sector uses only general capital to produce new capital. Pollution is assumed to be generated by the capital goods that are used only in the production of consumption goods. By contrast, our model incorporates human capital that is different from the physical capital in pollution physical capital. We note that since p(0)=1 (i.e., the price of general physical capital), and p(z) is increasing in z, it is more costly to use environmentally cleaner technology. In our model, the choice of cleaner technology implies that more physical capital is used for pollution abatement.[14] Therefore, given the existing stock of physical capital, a productive physical capital that is equipped with self-cleaning or pollution-abatement devices (e.g., mufflers for noise, bag houses for particulates, scrubbers, stacks, electrostatic precipitators for air pollution, water treatment plants, water recycling and conservation systems for water pollution, etc.) produces less output than the one that is used for only production without pollution abatement. Based on the price of each type physical capital good, the differentiated physical capital goods can be produced from the existing stock of general physical capital, k, according to the following constraint:

$$k = \int_0^\infty p(z)k(z)dz = \int_0^\infty (1+z)k(z)dz . \tag{3.1}$$

Although many differentiated physical capital goods can be used in the production of final output, only one type physical capital will be chosen at any given point in time. For example, if the government regulates pollution directly by setting the

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intensities, and plays an important role in generating growth in the presence of pollution.

[14] Using a neoclassical growth model, Gruver (1976) studied the optimal division of investment between pollution control capital and directly productive capital. He derived the optimal investment pattern from the shadow prices of the two types of capital. In our model, there is no distinction between pollution control capital and productive capital, and we do not focus on analyzing the optimal investment pattern between these two types of capital. However, our model implicitly assumes that physical capital is used for pollution abatement by the choice of production technology using a cleaner physical capital. A differentiated physical capital, which is used in a production process in our model, can be interpreted as the one incorporating both pollution abatement capital and directly productive capital.

emission standard such that any physical capital of a type below $\bar{z}>0$ cannot be used[15], then only $\bar{z}$ type capital will be used by firms to maximize the output, subject to the constraint on the available types of physical capital, k(z), $z \in [\bar{z}, \infty)$. Assuming that only one type physical capital good is used in production at time t, say k(z(t)), then from equation (3.1), the total amount of a physical capital good of type z(t) is given as

$$k(z(t)) = \left(\frac{1}{1+z(t)}\right)k(t), \tag{3.2}$$

where k(t) is the existing stock of general capital at time t. For analytical convenience, we assume that we can produce any specific type of differentiated physical capital good from general physical capital at any point in time with no extra cost.

The final output is produced using human capital and differentiated physical capital, according to the following production function:

$$y(t) = k(z(t))^{\alpha}\left((1-u(t))h(t)\right)^{1-\alpha} = \left(\frac{1}{1+z(t)}\right)^{\alpha} k(t)^{\alpha}\left((1-u(t))h(t)\right)^{1-\alpha}, \tag{3.3}$$

$$0 < \alpha < 1, \quad z(t) \geq 0,$$

[15] Direct control is the most common method of pollution control in the U.S.. For example, Freon refrigerators and leaded gasoline using engines are no longer produced for household consumption. Also, regulation on the equipment that can be used by commercial fishing fleets, or that on the use of pesticide in agriculture is an example in the natural resource industry.

where $k(z(t))$ is the differentiated physical capital of type $z(t)$, and $k(t)$ is the stock of general physical capital at time t. A Cobb-Douglas relationship exists between human capital and differentiated physical capital in the final output production function given in (3.3). However, the differentiated physical capital input is divided into two factors – general physical capital, $k(t)$, and the quality level of differentiated physical capital, $z(t)$. Therefore, the production technology exhibits constant returns to scale in human capital, $h(t)$, and general physical capital, $k(t)$, but is decreasing in the quality level of differentiated physical capital, $z(t)$. This implies that given the existing stock of general physical capital, using a cleaner physical capital in production reduces the output level.

While a certain type of differentiated physical capital is used in production of the final output, the produced output is used for the purpose of accumulating general physical capital as well as consumption. Assuming that physical capital doesn't depreciate, the stock of general physical capital, $k(t)$, evolves according to the following standard accumulation equation:

$$\dot{k}(t) = y(t) - c(t), \tag{3.4}$$

where $c(t)$ denotes consumption at time t.

The second sector is a pure investment sector because the output is not used for consumption but only for the purpose of accumulating human capital which also can be broadly interpreted as intellectual capital or technological knowledge. Assuming that human capital does not depreciate, human capital is produced using only human capital, according to the linear technology as in Lucas (1988):

$$\dot{h}(t) = \delta u(t)h(t), \tag{3.5}$$

where $\delta > 0$ is a productivity parameter and u(t) is the fraction of one unit of non-leisure time devoted to human capital accumulation at time t. As in Lucas (1988), Rebelo (1991), and Caballé and Santos (1993), human capital accumulation increases effective labor, and a higher effective labor raises the productivity of physical capital. Also, an increase in human capital raises the worker's wage per unit of time, which equals the marginal product of human capital in the final output production, multiplied by the current level of human capital. Because human capital is accumulated by the process of education, training, and research and development, the technology of human capital accumulation is relatively clean compared to the production technology of final output. So, we assume that the process of producing human capital does not generate pollution at all, so pollution does not affect the marginal productivity of human capital in its production.

3.2 Pollution

Since pollution[16] can be defined as any stock or flow of physical substance that impairs our physical or mental capacity to enjoy life—harms on human health or damage to the amenity value of the environment—it enters the instantaneous utility function with a negative marginal utility.

Pollution is assumed to have positive marginal product in the production side because any increase in pollution allowed in the production process can release resources from pollution abatement to produce more output.[17] Moreover, in the absence of pollution control and pollution abatement efforts, potential output of the economy can be achieved by using the dirtiest technology (e.g., Copeland and Taylor (1994), Jones and Manuelli (1995), and Stokey (1998)). In this sense, pollution in the production side can be defined as a joint product or "undesirable" but "inevitable" byproduct caused by final output production. Also, because of the positive marginal productivity of pollution, some analytical papers in this literature treat pollution as a normal input of production (e.g., Pittman (1981), Tahvonen and Kuuluvainen (1993), López (1994), and Bovenberg and Smulders (1995)).

To make a contribution to the literature on growth and the environment, we present a unique and detailed specification of pollution generating process in our model, which captures a realistic feature in the nature and source of pollution. Basically, we assume that pollution is generated in the production process of final output. In contrast

[16] Here, we follow the definition of pollution by Keeler, Spence and Zeckhauser (1972,1977). When pollution is more broadly defined, it is also referred to as the extractive use of the environment, or the intense exploitation of the resource such as deforestration (López (1994), Bovenberg and Smulders (1995)).

[17] This may not be true when pollution has its impact as a stock on production. Environmental degradation can affect individual welfare indirectly by reducing the productivity in the respective industry. Also see

with the previous studies dealing with pollution, in which total pollution is simply proportional to output (e.g., Keeler et al (1972), Gruver (1976), John and Pecchenino (1994), Ligthart and Ploeg (1994),Copeland and Taylor (1994) and Elbasha and Roe (1996)), or to the pollution-generating inputs (e.g., Forster (1972), Hung, Chang, and Blackburn (1994), Selden and Song (1995), Jones and Maunelli (1995) and Byrne (1997)), we make a distinction from those by assuming that pollution level depends on the type of differentiated physical capital used in production and the production technology as well as the output level.

Let $q(z(t)) = \left(\frac{1}{1+z(t)}\right)^{\beta}$, where $\beta > 1$, $z(t) \geq 0$, denote the pollution generating level per unit of type z(t) physical capital used in production at time t. From the above expression of q(z(t)), we see that q(0) = 1, and q(z(t)) is decreasing and convex in z(t), so higher quality physical capital generates less pollution per unit. Also, we assume that pollution intensity of production differs depending on the share of using more polluting inputs in the total cost of production, which can be represented by the production technology parameter α.

The specification of pollution generating process is given by

$$x(t) = \left(q(z(t))k(z(t))\right)^{\alpha}\left((1-u(t))h(t)\right)^{1-\alpha} = \left(\frac{1}{1+z(t)}\right)^{\alpha(\beta+1)} k(t)^{\alpha}\left((1-u(t))h(t)\right)^{1-\alpha}, \quad (3.6)$$

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the footnote 19 for examples.

where x(t) is the pollution generated per individual production at time t. From (3.3) and (3.6), we can derive the relationship between pollution and output in terms of the type of physical capital used in production as below:

$$\frac{x(t)}{y(t)} = \left(\frac{1}{1+z}\right)^{\alpha\beta} > 0. \tag{3.7}$$

Pollution is proportional to output, but the ratio of pollution relative to output is decreased as the cleaner (i.e., higher z) type of physical capital is used in production, and the production technology is less physical capital intensive (i.e., less α). From the above formulation, we see that pollution can be measured in the same units of output. Our research objective is not to estimate the exact amount of pollution in the correct measure of units, but to observe the change in pollution level with respect to the change in other variables of the economy. Thus, we will not be in pursuit of studying how to measure the pollution in both quantitative and qualitative dimensions in this paper.[18]

We may choose different methods of pollution control that will result in different outcomes depending on the nature of pollution, so it is also necessary to make a distinction between flow and stock effects of pollution[19]. In Chapter 4, we treat pollution as a flow that is negatively related to marginal utility but has a positive marginal product. In Chapter 5, we will extend the basic model to examine whether we have different

[18] See D'Arge and Kogiku (1973) for the study on this matter.
[19] Pollution as a flow has a positive marginal product in most cases (e.g., noise pollution, air or water pollution in the manufacturing industries). However, a gradual increase in pollution as a stock will eventually decrease productivity in the respective industries (e.g., air and soil quality in agricultural

results implications about the long-run behavior of the economy when pollution is treated as a stock.

3.3 Preferences

The intertemporal utility of a representative consumer is defined as

$$\int_0^{\infty} e^{-\rho t} U(c(t), x(t)) dt \ , \tag{3.8}$$

where ρ represents the rate of time preference. U(c(t), x(t)) is the instantaneous utility function which represents an individual consumer's preferences over consumption and pollution at time t, and takes the following form:

$$U(c(t), x(t)) = \upsilon(c(t)) - v(x(t)) = \frac{c(t)^{1-\sigma}}{1-\sigma} - \frac{\phi x(t)^{\gamma}}{\gamma}, \quad \sigma > 0,\ \gamma > 1,\ \phi > 0. \tag{3.9}$$

Since we assume that the utility function is additively separable in consumption and pollution, an increase in consumption will not affect the marginal disutility from pollution, and vice versa (i.e., $U_{cx} = 0$). Note that preferences expressed in the utility

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industry, water quality in fishery and deforestration).

function in (3.9) are non-homothetic[20] in consumption and pollution. This is in contrast to the previous studies in this literature in which preferences are assumed to be homothetic (e.g., Bovenberg and Smulders (1995), Elbasha and Roe (1995, 1996), and Byrne (1997)). The assumption of homothetic preferences implies unitary income elasticity with respect to the environmental goods. In the early stage of economic development, per capita income is relatively low and so is the environmental cost of production, so people are concerned more about economic growth than about environmental preservation. In this case, income and pollution are likely to increase simultaneously. However, as a per capita income level is increased, environmental quality will become a scarce and luxurious good as a result of the bias toward growth. In this respect, the homotheticity assumption may not be appropriate to represent our preferences toward consumption and environmental quality as per capita income changes.

One more thing to note about the utility function given in (3.9) is that ϕ could be a function of population density in a more general model. In such a model, we could examine how the damage caused by total pollution in the economy affects an individual consumer's utility. The impact of total pollution on individual's utility depends not only on the pollution generated per individual but also on the population density. While an individual generates pollution by participating in productive activities, he/she is also affected by the pollution generated by others of a certain local community. As

[20] López (1994) assumed non-homothetic preferences in order to derive the inverted U-shaped relationship between pollution and income using a neoclassical model. Also, he assumed that the coefficient of relative risk aversion changes as income level changes. He argued that if preferences are homothetic, the increase in output will necessarily cause the level of pollution, even if optimally controlled, to proportionally increase. However, non-homotheticity assumption can be a necessary but not sufficient condition to derive the inverted U-shaped relationship between pollution and income in our model.

population density increases, individuals are harmed more by pollution generated by others, and ϕ will be greater.[21] However, since we assume that the size of population is constant, $\phi > 0$ is taken to be constant for analytical convenience.

[21] In most of the papers in this literature, pollution enters the individual utility function as a per capita term, which implicitly assumes that people are harmed by their own pollution only. Furthermore, Copeland and Taylor (1994) assume that population size will lessen the impact of total pollution on individual utility by treating the population size as the physical size of the economy. However, if the population size matters, we basically assume that the individual is affected by the total amount of pollution that increases as more people participate in production in a local area.

Chapter 4

Theoretical Analysis of an Inverted U-Shaped Relationship between Growth and Pollution

The main objective in this chapter is to present a simple theoretical model that is consistent with the empirical evidence of an inverted U-shaped relationship between per capita income and pollution levels, in both a static and dynamic setting. Most previous theoretical studies investigating the income-pollution relationship are focused on either a static analysis or a long-run growth analysis. Using a theoretical model that incorporates environmental externalities, our goal is to provide a theoretical basis for an inverted-U pattern in both static and dynamic settings. Theoretical investigation on the relationship between growth and pollution will answer the questions left open in the empirical studies.

In the following sections, we will analyze the models given in this chapter as a social planner's problem. Especially for a dynamic model, the social planner will maximize the intertemporal utility give in (3.8), subject to the accumulation equations of physical and human capital and to the resource constraint of the existing stock of general capital.

The remainder of this chapter is organized as follows. In Section 4.1 we present a simple static model to analyze the relationship between income and environmental degradation in the static stage. An inverted U-shaped relationship between per capita income and pollution is derived from the static model of pollution control. Also, an alternative model, in which pollution is treated as a normal input of production, is presented to explore the underlying implications of the inverted U-shaped relationship between per capita income and pollution. Section 4.2 examines the dynamic path of pollution during the transition of the economy by using the basic model described in Chapter 3, which can be compared with the empirical findings obtained from the time-series data.

4.1 Static Analysis of the Relationship between Income and Pollution

4.1.1 A Static Model of Pollution Control

The interaction between income and pollution has been extensively examined in empirical studies[22] using cross-sectional as well as time-series data. One of the main objectives in this paper is to investigate the relationship between pollution and the per capita income level in a theoretical framework. In this section we will simplify the basic model presented in the previous chapter into a static one in order to focus on

[22] As we review the existing literature in Chapter 2, the early empirical studies focused on the negative effect of environmental regulation on the productivity and growth (e.g., Gollop and Roberts (1983), Jorgenson and Wilcoxen (1990, 1992)). Recently, however, substantial empirical evidence has shown that environmental degradation and income have an inverted U-shaped relationship (e.g., World Bank Development Report (1992), Grossman and Krueger (1993, 1995), Selden and Song (1994), Holtz-Eakin and Selden (1995), Carson and Jeon (1997), and Hilton and Levinson (1998)).

investigating how different levels of per capita income affect the level of pollution in a static setting.

Using the specification of final output production and the pollution generating process given in (3.3) and (3.6), we can derive the direct relationship between pollution and per capita income level as

$$x = \left(\frac{1}{1+z}\right)^{\alpha\beta} y. \tag{4.1}$$

From (4.1), we see that the only variable that determines the change in the relationship between x and y is z, a quality index of the differentiated physical capital used in production. In a dynamic model, the optimal choice of z and its dynamic behavior are affected by the optimality conditions for other related variables as well. However, the simplest way to find an intuitive relationship between income and pollution is to analyze a change in z in a static framework by assuming that z is the only choice variable with all other variables held constant.[23] For the convenience of static analysis, we assume that all output is consumed and all non-leisure time is devoted to final output production, i.e., u = 0.[24] Hence, the simplified version of per capita consumption and pollution in a static framework can be specified as

[23] In this section, z will be treated as the only choice variable because the change in x with respect to y is directly affected by only z. A more general analysis of z(t) with respect to the change in other related variables over time will be given in a dynamic analysis in the next section.

[24] Since there is no accumulation for either physical or human capital, this is a one sector static model. In

$$c = y = \omega^{\alpha} y_p , \tag{4.2}$$

$$x = \omega^{\alpha(\beta+1)} y_p . \tag{4.3}$$

where $\omega \equiv \frac{1}{1+z}$, $z \geq 0 \Rightarrow \omega \in (0,1]$. We define the potential output of the economy, $y_p = k^{\alpha}h^{1-\alpha}$, as the economy's maximum output capacity that could be achieved by using only the dirtiest (general) physical capital good in the final output production (i.e., $z = 0 \Rightarrow \omega = 1$). Since we assume that $\beta > 0$, x is an increasing and convex function of y for fixed y_p. Note that as long as ω is constant (e.g., when $\omega = 1$ before the transition), the pollution level, x, will proportionally increase with an increase in y (or y_p).

Substituting (4.2) and (4.3) into the instantaneous utility function of the representative consumer, we can write the social planner's problem as

$$\max_{\omega\in(0,1]} \frac{(\omega^{\alpha} y_p)^{1-\sigma}}{1-\sigma} - \frac{\phi\left(\omega^{\alpha(\beta+1)} y_p\right)^{\gamma}}{\gamma}, \quad \sigma > 0, \quad \gamma > 1 . \tag{4.4}$$

For the fixed potential output y_p, the optimal quality level of differentiated physical capital, $\omega^*(y_p)$, is given by

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this case, human capital can be interpreted as a labor input available in the economy.

$$\omega^*(y_p) = 1 \qquad \text{if} \quad y_p \leq y_c, \tag{4.5a}$$

$$\omega^*(y_p) = \left[\frac{y_c}{y_p}\right]^{\frac{\gamma-1+\sigma}{\alpha((\beta+1)\gamma-1+\sigma)}} \qquad \text{if} \quad y_p > y_c, \tag{4.5b}$$

where y_c is defined as the critical level of output given by

$$y_c = \left(\frac{1}{\phi(\beta+1)}\right)^{\frac{1}{\gamma-1+\sigma}}. \tag{4.6}$$

If the potential output level is less than or equal to y_c, only the dirtiest physical capital is used, i.e., $\omega^*=1$. For potential output greater than y_c, it is optimal to control pollution by using a higher quality (i.e. cleaner but more expensive) physical capital, and the optimal quality level is given in (4.5b).[25] Also, as we can see from (4.5b) the optimal emission standard, which can be represented by the quality level of differentiated physical capital, ω(or z), becomes increasingly stringent as the potential output level increases (i.e., $\omega^{*\prime}(y_p) < 0$ if $y_p > y_c$). Our static model is similar to Stokey (1998) in the sense that the optimal control of pollution depends on the size of potential output. However, she treats

[25] When y_p is greater than y_c, the second-order condition for a maximum is also satisfied. If we define $U(\omega)$ as $U(\omega) = \frac{(\omega^\alpha y_p)^{1-\sigma}}{1-\sigma} - \frac{\phi\left(\omega^{\alpha(\beta+1)} y_p\right)^\gamma}{\gamma}$, and ω^* satisfies $U'(\omega^*) = 0$, then we see that

consumption goods and pollution as joint products of a single input. By contrast, we assume that the existing stocks of physical and human capital determine the size of potential output, and that pollution is controlled through the choice of differentiated physical capital in production. Hence, our model yields different results and implication from those of Stokey (1998) in the strictness of pollution control or the critical level of income.[26]

Based on the optimal level of physical capital's quality, which is determined by the potential output level, the optimal levels of per capita consumption and pollution can be expressed in terms of potential output as below:

$$c^*(y_p) = y^*(y_p) = y_p, \qquad \text{if} \quad y_p \le y_c, \tag{4.7a}$$

$$x^*(y_p) = y_p \qquad \text{if} \quad y_p \le y_c, \tag{4.7b}$$

$$c^*(y_p) = y^*(y_p) = y_c^{\frac{\gamma-1+\sigma}{(\beta+1)\gamma-1+\sigma}} y_p^{\frac{\beta\gamma}{(\beta+1)\gamma-1+\sigma}}, \quad \text{if} \quad y_p > y_c, \tag{4.7c}$$

$U''(\omega^*) = [(1-\sigma) - \gamma(\beta+1)]\alpha^2 \omega^{*\alpha(1-\sigma)-2} y_p^{1-\sigma} < 0$ because $[(1-\sigma) - \gamma(\beta+1)] < 0$.

[26] In our model, the greater is the share of differentiated physical capital in the final output production, with the less stringent regulation we can reduce pollution. Furthermore, if we assume that the pollution generating level per unit of human capital is $0 < \varphi < 1$ (although we normalize φ to one for analytical convenience), then the critical level of income becomes $\left(\frac{1}{\phi(\beta+1)\varphi^{(1-\alpha)\gamma}}\right)^{\frac{1}{\gamma-1+\sigma}}$. In this case, the greater is the share of differentiated physical capital in the final output production, the lower the critical level of income is. Estimating a critical level of income has also been a major concern of interest in the empirical literature (e.g., Grossman and Krueger (1993, 1995), Selden and Song (1994)). So we make a contribution by providing theoretical backgrounds to explain why the critical level of income could be different depending on the different production technologies.

$$x^*(y_p) = y_c^{\frac{(\beta+1)(\gamma-1+\sigma)}{(\beta+1)\gamma-1+\sigma}} y_p^{\frac{\beta(1-\sigma)}{(\beta+1)\gamma-1+\sigma}} \qquad \text{if } y_p > y_c, \qquad (4.7d)$$

where y_c is as before, i.e., $y_c = \left(\frac{1}{\phi(\beta+1)}\right)^{\frac{1}{\gamma-1+\sigma}}$. Since we are interested in finding the direction of change in pollution with respect to the change in per capita income levels, we can derive the relationship between pollution and per capita income directly from (4.7a) through (4.7d) by removing y_p from the expressions of $c^*(y_p)\,(= y^*(y_p))$ and $x^*(y_p)$:

$$x^* = y^* \qquad \text{if } y_p \le y_c, \qquad (4.8a)$$

$$x^* = y_c^{\frac{\gamma-1+\sigma}{\gamma}}\, y^{*\frac{1-\sigma}{\gamma}} = \left(\frac{1}{\phi(\beta+1)}\right)^{\frac{1}{\gamma}} y^{*\frac{1-\sigma}{\gamma}} \qquad \text{if } y_p > y_c. \qquad (4.8b)$$

Equations (4.8a) and (4.8b) describe the relationship between per capita income and pollution from a social planner's point of view. If the potential output capacity of the economy is not big enough to produce more than the critical level of output, there is no pollution control and only the dirtiest physical capital goods will be used to produce the maximum output. Accordingly, pollution proportionally increases as actual per capita income level increases for $y_p \le y_c$. However, once the economy's production capacity reaches a critical output level, it is optimal to control pollution by choosing to use cleaner physical capital goods at the expense of less than potential output produced. In this case, the direction of change in pollution with respect to the change in actual per capita income depends on the elasticity of the marginal utility of consumption. For $y_p > y_c$, pollution increases, decreases, or is constant as actual per capita income level increases if σ is less than, greater than, or equal to one, respectively. Figure 4.1 illustrates the three possible relationships between per capita income and pollution, depending on the value of σ. An inverted U-shaped relationship between per capita income and pollution is derived if σ is greater than one.

4.1.2 An Alternative Model of Pollution as an Input

The intuition behind the results derived from the static model of pollution control in Section 4.1.1 comes out more clearly if we modify the model into the one in which pollution is treated as a normal input. Also, a more general analysis is possible if we express the utility and production functions in a more generalized functional form. We can show that a production function with pollution inputs is directly derived from equations (4.2) and (4.3). First, from equation (4.3), a quality index of differentiated physical capital (ω) can be expressed in terms of pollution (x) and potential output (y_p):

$$\omega = x^{\frac{1}{\alpha(\beta+1)}} y_p^{\frac{1}{\alpha(\beta+1)}}. \qquad (4.9)$$

Second, substitute (4.9) into (4.2), then we get

$$c = y = x^{\frac{1}{\beta+1}} y_p^{1-\frac{1}{\beta+1}} . \tag{4.10}$$

Equation (4.10) implies that the actual output is in effect produced with pollution and potential output as inputs, and the production technology exhibits constant returns to scale to x and y_p. Also, from (4.10), we see that no output is produced without pollution input, and that actual output is increased with more pollution input. However, actual output cannot be increased indefinitely by increasing only pollution input because there is an upper bound in the use of pollution input. Since the quality index of differentiated physical capital, ω, is bounded below by one, pollution cannot exceed its maximum level when $\omega = 1$, i.e., $0 < x \leq y_p$. Some authors who simply treat pollution as a normal input of production in their models do not take into account this kind of upper bound in the use of pollution input (e.g., Tahvonen and Kuuluvainen (1993), López (1994), and Bovenberg and Smulders (1995)). They argued that producers would select an infinitely high level of pollution to produce the maximum output if there were no government intervention in a market economy. Since pollution by itself cannot be a productive input, they ignored an important restriction on the feasible production technology in the sense that the actual output cannot increase with pollution beyond the potential output level.

By using (4.10), the social planner's problem given in (4.4) can be rewritten as

$$\max_{x\in[0,y_p]} \frac{\left(x^{\frac{1}{\beta+1}} y_p^{1-\frac{1}{\beta+1}}\right)^{1-\sigma}}{1-\sigma} - \frac{\phi x^{\gamma}}{\gamma}, \quad \sigma>0, \ \ \gamma>1. \tag{4.11}$$

Given the potential output level, the optimal level of pollution input, $x^*(y_p)$, should satisfy

$$x^*(y_p) = y_p \ \text{ if } \ \frac{1}{\beta+1}\left(y_p^{1-\frac{1}{\beta+1}}\right)^{1-\sigma} x^*(y_p)^{\frac{1-\sigma}{\beta+1}-1} > \phi x^*(y_p)^{\gamma-1}, \tag{4.12a}$$

$$\frac{1}{\beta+1}\left(y_p^{1-\frac{1}{\beta+1}}\right)^{1-\sigma} x^*(y_p)^{\frac{1-\sigma}{\beta+1}-1} = \phi x^*(y_p)^{\gamma-1} \quad \text{ if } \ x^*(y_p) < y_p. \tag{4.12b}$$

If we solve the above conditions for an optimum, the critical level of output and the optimal levels of consumption and pollution are exactly the same as those obtained from the previous model in Section 4.1.1 (i.e., (4.6) and (4.7a-d)).

Furthermore, for a more general analysis, the social planner's problem given in (4.11) can be expressed in a more generalized functional form:

$$\max_{x\in[0,y_p]} U(c, x) = U(f(x, y_p), x), \tag{4.13}$$

where U is the instantaneous utility function that depends on consumption, c, and pollution, x, and f is a production function of consumption goods using pollution input given the potential output level. We assume that f is linearly homogeneous in y_p and x, and $f_x > 0$, $f_{xx} < 0$, $f_{y_p} > 0$, $f_{y_p y_p} < 0$, and that U is additively separable in c and x, and $U_c > 0$, $U_{cc} < 0$, $U_x < 0$, $U_{xx} < 0$.

Given the potential output level, y_p, solving the first-order condition for a maximum gives the optimal level of pollution as

$$x^* = y_p \quad \text{if} \quad U_c f_{x^*} + U_{x^*} > 0, \tag{4.14a}$$

$$U_c f_{x^*} + U_{x^*} = 0 \quad \text{if} \quad x^* < y_p. \tag{4.14b}$$

The interpretation of the above conditions for the optimal level of pollution is as follows. $U_c f_{x^*}$ is the marginal benefit from increased output by allowing more pollution to be generated and U_{x^*} is the marginal cost of higher pollution, so the sum of $U_c f_{x^*}$ and U_{x^*} represents the net marginal benefit of higher pollution. If the net marginal benefit from increasing pollution is always positive, there is no need to control pollution and we allow the maximum level of pollution to be generated. If the net marginal benefit from allowing more pollution becomes negative after pollution reaches a certain level, we find the optimal level of pollution at which the net marginal benefit of pollution is zero. In the case of a corner solution of x in (4.14a), it is clear that pollution is increased with actual output.[27] In order to analyze the effect of y_p on x in the case of an interior solution, form the differential of (4.14b) with respect to x and y_p:

$$(U_{cc}f_x^2 + U_c f_{xx} + U_{xx})dx + (U_{cc}f_{y_p}f_x + U_c f_{xy_p})dy_p = 0. \tag{4.15}$$

The terms in the first parenthesis on the left-hand-side of (4.15) should be negative by the second-order condition for the optimal solution of the social planner's problem given in (4.13). Hence, the sign of $\frac{dx}{dy_p}$ depends on the sign of the terms in the second parenthesis on the left-hand-side of (4.15), i.e., $\frac{dx}{dy_p} \gtreqless 0$ as $U_{cc}f_{y_p}f_x + U_c f_{xy_p} \gtreqless 0$.

Using c = y = f (y_p, x), we can rewrite the above condition as

$$\frac{dx}{dy_p} \gtreqless 0 \text{ as } \frac{f_{y_p x}f}{f_{y_p}f_x} \gtreqless -\frac{U_{cc}c}{U_c}. \tag{4.16}$$

Note that the left-hand-side of the second inequality in (4.16) is the reciprocal of the elasticity of substitution in production between pollution and non-pollution conventional inputs[28], σ_y. The right-hand-side of the second inequality in (4.16) is the elasticity of the

[27] Even if the actual output (f) is different from the potential output level (y_p), the direction of change in x with respect to y_p is the same as the direction of change in x with respect to f because $f_{y_p} > 0$.

[28] Potential output (y_p) can be interpreted as all conventional factors in the economy in the sense that they represent the economy's production capacity. Also, economic growth can be represented by the expansion of those conventional factors.

marginal utility of consumption, σ_c. Thus, the direction of change in x with respect to y_p depends on the relative magnitude of σ_y and σ_c. The higher the elasticity of substitution in production between pollution and conventional inputs, and the higher the elasticity of the marginal utility of consumption, the pollution level is more likely to decrease with economic growth.

The intuition behind these results is as follows. The elasticity of the marginal utility of consumption (σ_c) shows how the marginal utility of consumption declines as we consume more. Thus, as σ_c is higher, the price of pollution (the social marginal cost of pollution) increases faster as the consumption (income) level increases.[29] In other words, a higher level of σ_c implies that we are willing to give up more consumption to reduce the given amount of pollution as our income level increases. On the other hand, a higher elasticity of substitution in production between pollution and conventional inputs implies that firms are willing to use more conventional inputs instead of reducing pollution input in response to an increase in the price of pollution input. That is, as σ_y is higher, even a small increase in the price of pollution will induce firms to substitute conventional inputs for pollution by a larger extent, so the pollution generating level per unit of conventional inputs will be reduced by a larger extent. As a consequence, if both σ_c and σ_y are high enough for σ_c to be greater than $1/\sigma_y$, the price of pollution will be

[29] If we let the price of pollution (P_x) be endogenously determined to reflect the true social marginal cost of pollution, the price of pollution is derived from the first-order condition for the optimal level of pollution given in (4.14b): $P_x = f_x = -\frac{U_x}{U_f} = -\frac{U_x}{U_c}$.

sharply increased with income, and firms are willing to reduce pollution by a large extent in response to the higher price of pollution.

In this case, we can derive an inverted U-shaped relationship between income and pollution. The pollution level increases with income if the per capita income level is below the critical level of income given in (4.6). However, pollution decreases with income that exceeds the critical level if $\sigma_c > 1/\sigma_y$ (or $\sigma_c > 1$ in the case of $\sigma_y = 1$). Some empirical results indicate that the elasticity of the marginal utility of consumption lies between one and two, or as an alternative measure, the intertemporal elasticity of substitution in consumption is less than one.[30] These empirical results support the theoretical background for the downward sloping part of an inverted U-shaped relationship between per capita income and pollution.

4.2 Dynamic Behavior of the Optimal Pollution Level

In this section, potential output is not given but endogenously determined by the existing stocks of physical and human capital that are accumulated according to their accumulation equations given in (3.4) and (3.5), respectively. We are interested in the time paths of the actual output and pollution in order to examine whether they are also consistent with the inverted U-shaped relationship between growth and pollution, which has been empirically confirmed using the time-series data (e.g., Carson and Jeon (1997)).

So we use the dynamic model to focus on analyzing the qualitative behavior of the optimal pollution control and the corresponding level of pollution over time. Unlike the static model, the direct relationship between per capita income and pollution cannot be obtained from the dynamic model. We solve for the optimal level of pollution in terms of other variables and parameters. If the variables that determine the level of pollution are increasing (or decreasing) over time at constant rates, the dynamic behavior of the optimal pollution level can be easily obtained by substituting the growth rates of those variables into the growth rate of pollution. Hence, we can investigate the dynamic relationship between per capita income and pollution over time by solving for the growth rate of pollution in terms of per capita income.

As in the previous section, we consider the social planner's problem. The social planner will choose time paths for consumption, c(t), quality of differentiated physical capital for pollution control, z(t), and a fraction of non-leisure time devoted to human capital accumulation, u(t), to maximize the lifetime utility of an infinitely lived representative individual:

$$\max \int_0^\infty e^{-\rho t}\left[\frac{c^{1-\sigma}(t)}{1-\sigma}-\frac{\phi x(t)^\gamma}{\gamma}\right]dt \qquad (4.17)$$

$$\text{s.t.} \quad \dot{k}(t) = \omega^\alpha(t)k^\alpha(t)\big((1-u(t))h(t)\big)^{1-\alpha} - c(t),$$

$$\dot{h}(t) = \delta u(t)h(t),$$

$$x(t) = \omega^{\alpha(\beta+1)}(t)\, k^\alpha(t)\big((1-u(t))h(t)\big)^{1-\alpha},$$

[30] For example, see Hansen and Singleton (1983), Hall (1988), and Epstein and Zin (1991).

$$\omega(t) = \frac{1}{1+z(t)},\ z(t) \geq 0,\ \omega(t) \leq 1,$$

where all notations are the same as those described in the basic model in Chapter 3, and the initial stocks of physical and human capital are given as $k(0) = k_0$ and $h(0) = h_0$, respectively.[31]

The current value Hamiltonian for the social planner's problem is

$$H = \frac{c^{1-\sigma}}{1-\sigma} - \frac{\phi}{\gamma}\left[\omega^{\alpha(\beta+1)}k^{\alpha}\left((1-u)h\right)^{1-\alpha}\right]^{\gamma} + \lambda_1\left[\omega^{\alpha}k^{\alpha}\left((1-u)h\right)^{1-\alpha} - c\right] + \lambda_2\delta uh + \mu(1-\omega),$$

(4.18)

where λ_1 and λ_2 denote the costate variables associated with the accumulation of physical and human capital, respectively, and μ is a Lagrange multiplier resulting from the inequality constraint that $z \geq 0 \Rightarrow \omega \leq 1$. The Kuhn-Tucker condition implies that the optimal values of μ and ω must meet the following condition: $\mu(1-\omega) = 0$; therefore,

$$\omega < 1 \Rightarrow \mu = 0,\ \mu \geq 0 \Rightarrow \omega = 1. \tag{4.19}$$

[31] Appendix A shows the detailed process of how we derive the optimal solutions for the social planner's problem given in (4.17).

Solving the first-order conditions with respect to the control variables c and ω, and using (4.19), we get the optimal paths of consumption and the quality of differentiated physical capital over time as below:

$$c^{-\sigma} = \lambda_1, \tag{4.20}$$

$$\omega = \begin{cases} 1, & \text{if } \lambda_1 \geq \frac{1}{\psi}\left[k^{\alpha}((1-u)h)^{1-\alpha}\right]^{\gamma-1}, \quad (4.21a) \\ \left[\psi\lambda_1\left[k^{\alpha}((1-u)h)^{1-\alpha}\right]^{-\gamma}\right]^{\frac{1}{\alpha(\gamma(\beta+1)-1)}}, & \text{if } \lambda_1 < \frac{1}{\psi}\left[k^{\alpha}((1-u)h)^{1-\alpha}\right]^{\gamma-1}, \quad (4.21b) \end{cases}$$

where ψ is a predetermined function of the parameters of production technology and preference, defined as $\psi \equiv \frac{1}{\phi(\beta+1)}$. Equation (4.20) implies that the marginal utility of consumption is equal to the shadow value of physical capital all the time for the optimal path of consumption. However, equations (4.21a) and (4.21b) imply that the optimal quality of differentiated physical capital is divided into the corner and interior solutions depending on whether the inequality constraint is binding or not. More specifically, the optimal strategy for pollution control, represented by ω, depends on the level of potential output, defined as $y_p(t) = k(t)^{\alpha}((1-u(t))h(t))^{1-\alpha}$, relative to the shadow value of physical capital, $\lambda_1(t)$. It is clear from (4.20) that the shadow value of physical capital falls if consumption rises over time:

$$\frac{\dot{\lambda}_1}{\lambda_1} = -\sigma\frac{\dot{c}}{c} < 0. \tag{4.22}$$

In this case, there is a critical point of time in the evolution of λ_1, defined as τ such that $\lambda_1(\tau) = \frac{1}{\psi}\left[k(\tau)^{\alpha}\left((1-u(\tau))h(\tau)\right)^{1-\alpha}\right]^{\gamma-1}$, before which there is no pollution control at all, and after which pollution should be optimally controlled. In other words, if the economy's current level of potential output is relatively low, it is optimal not to control pollution but to use the dirtiest physical capital to maximize output. However, if the potential output grows enough to reach the critical level of output, defined as $y_c(t) = (\psi\lambda_1(t))^{\frac{1}{\gamma-1}}$[32], it is optimal to control pollution by using the cleaner physical capital in the final output production at the cost of fewer output produced. Hence the economy has a transition from the initial growth path in which $\omega = 1$, to the asymptotic long-run growth path in which $\omega < 1$. Once the potential output exceeds the critical level, the strictness of pollution control along the asymptotic long-run growth path can be obtained by taking logarithms and differentiating the equation (4.21b) with respect to time, t:

[32] In the static model, the critical level of income y_c, with which the economy's potential income level is compared to determine whether or not to control pollution, is fixed in terms of the related parameters. Here, the critical level of income $y_c(t)$ depends not only on the parameters that are the same as those in the static model, but also on the current shadow value of physical capital that varies over time.

$$\frac{\dot{\omega}}{\omega}=\frac{1}{\alpha(\gamma(\beta+1)-1)}\left[\frac{\dot{\lambda}_1}{\lambda_1}+(1-\gamma)\left(\alpha\frac{\dot{k}}{k}+(1-\alpha)\left(\frac{\dot{h}}{h}-\left(\frac{u}{1-u}\right)\frac{\dot{u}}{u}\right)\right)\right]. \tag{4.23}$$

If we assume that consumption and the stocks of physical and human capital all grow at constant rates, and u is constant[33] along the asymptotic long-run growth path, then $\frac{\dot{\omega}}{\omega}$ must be negative because $\beta > 0$, $\gamma > 1$, and the sign of the terms in the brackets in (4.23) is negative. Since ω is inversely related to the quality of differentiated physical capital, z, in terms of cleanliness, the reduction of ω in a growing economy implies that the optimal pollution control should be increasingly stringent with economic growth.

Since we are interested in the qualitative behavior of pollution, substitute (4.21a) and (4.21b) into (3.6) to analyze the optimal pollution level during the transition to the asymptotic long-run growth path:

$$x=\begin{cases} k^{\alpha}((1-u)h)^{1-\alpha}, & \text{if} \quad \omega=1, \quad (4.24a) \\ \left[(\psi\lambda_1)^{\beta+1}\left[k^{\alpha}((1-u)h)^{1-\alpha}\right]^{\beta}\right]^{\frac{1}{\gamma(\beta+1)-1}}, & \text{if} \quad \omega<1. \quad (4.24b) \end{cases}$$

Taking logarithms and differentiating (4.24a) and (4.24b) with respect to time yields the growth rate of pollution:

[33] In the static model, we assumed that u = 0. In the dynamic model, u is not zero but a choice variable that is between 0 and 1. However, we assume that u is constant in order to analyze the economic behavior in the steady state.

$$\frac{\dot{x}}{x} = \begin{cases} \alpha\frac{\dot{k}}{k} + (1-\alpha)\left(\frac{\dot{h}}{h} - \left(\frac{u}{1-u}\right)\frac{\dot{u}}{u}\right), & \text{if} \quad \omega = 1, \qquad (4.25a) \\ \frac{1}{\gamma(\beta+1)-1}\left((\beta+1)\frac{\dot{\lambda}_1}{\lambda_1} + \alpha\beta\frac{\dot{k}}{k} + (1-\alpha)\beta\left(\frac{\dot{h}}{h} - \left(\frac{u}{1-u}\right)\frac{\dot{u}}{u}\right)\right), & \text{if} \quad \omega < 1. \qquad (4.25b) \end{cases}$$

Assume that consumption grows and the stocks of physical and human capital accumulate at constant rates, and u is constant over time, even though the growth rates of c, k, and h, and the fraction u can be different between before and after the transition. Then it is clear from (4.25a) that if the potential output level is smaller than the critical level of output so that $\omega = 1$, then pollution level increases with the increase in physical and human capital. However, if the potential output level passes the critical level and the economy begins to take action for pollution control (i.e., $\omega < 1$), the asymptotic long-run growth rate of pollution depends on the growth rates of k(t), h(t), and $\lambda_1(t)$. Because we assume $\beta > 0$ and $\gamma > 1$, the optimal pollution level decreases in the long run if and only if the sign of the terms in parentheses of equation (4.25b) is negative.

In order to analyze the dynamic behavior of pollution after the transition, we assume that the economy asymptotically approaches a steady-state growth path. We define the steady-state growth path as a path along which all the optimality conditions are satisfied and all the variables c, y, k, h, ω, and x grow at constant (not necessarily the same and possibly zero) rates while the fraction of non-leisure time devoted to human capital accumulation, u, is constant. In Appendix A, we show that along the asymptotic long-run (steady-state) growth path,

$$\frac{\dot{k}}{k}=\frac{\dot{y}}{y}=\frac{\dot{c}}{c}=\frac{1}{\sigma+\vartheta}(\delta-\rho)>0^{34}\text{, where } \vartheta=\frac{\sigma+\gamma-1}{(1-\alpha)\beta\gamma}>0, \tag{4.26}$$

$$\frac{\dot{h}}{h}=\left(1+\frac{\gamma-1+\sigma}{(1-\alpha)\beta\gamma}\right)\frac{\dot{y}}{y}>0. \tag{4.27}$$

Hence, from (4.22), (4.26), and (4.27), we see that the growth rate of pollution given in (4.25b) is negative if and only if $(1-\sigma)\left(\beta+\frac{\gamma-1}{\gamma}\right)g_y<0$, where g_y is the asymptotic long-run growth rate of output. Therefore, along the asymptotic steady-state growth path after the potential output level passes the critical level of output, pollution declines with economic growth if and only if $\sigma>1$. If the elasticity of the marginal utility of consumption is greater than one, the dynamic behavior of the optimal pollution level displays an inverted U-shaped pattern, while the per capita income is continuously growing over time. Figure 4.2 illustrates the optimal paths of per capita income and pollution over time in the case of $\sigma > 1$. Since the per capita income is monotonically

[34] We assume $\delta>\rho$ so that the growth rates of k and h are positive. Furthermore, if $\delta<\rho$, the optimal value of the fraction of non-leisure time devoted to human capital accumulation, u, is negative. In order to have a positive value of u, it must be the case that $\delta>\rho$.

increasing over time, we can derive the relationship between per capita income and pollution by matching a unique level of pollution to each income level. Regardless of the value of σ, pollution is first increasing with income in the early stage of economic growth until the economy's per capita income level reaches the critical level. However, after the per capita income level exceeds the critical level, pollution level is increasing, decreasing, or constant as income grows along the asymptotic long-run growth path, depending on the value of σ. Figure 4.3 displays the change in pollution level as income grows in the case of $\sigma > 1$. As in the static model, an inverted U-shaped relationship between per capita income and pollution is derived from the dynamic model if σ is greater than one.

The basic model framework we have developed in this chapter can be extended to study a number of additional related issues. To reconsider the growth-environment analysis more extensively, many possible extensions can be made from our basic model. In this chapter, for example, we treat pollution as a flow, but we can explore and discuss the implications when pollution has its effect as a stock variable. So far we consider the social planner's problem of how to deal with the optimal control of pollution, but we also have to deal with the critical question of what policy guidelines can implement the optimal growth paths in a decentralized economy. We will discuss these issues with the extension of our basic model in the next chapters.

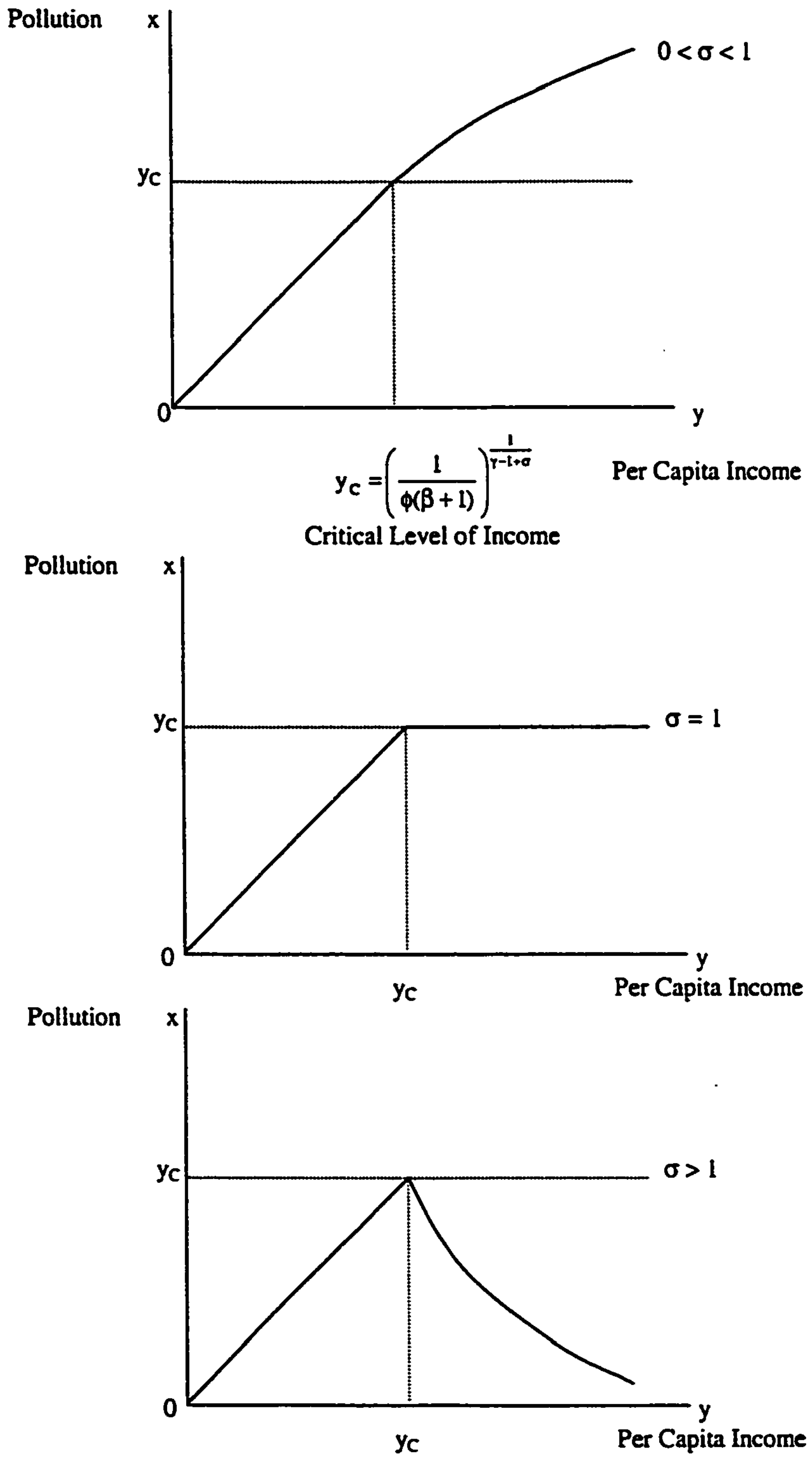

Figure 4.1: Per Capita Income and Pollution in the Static Model

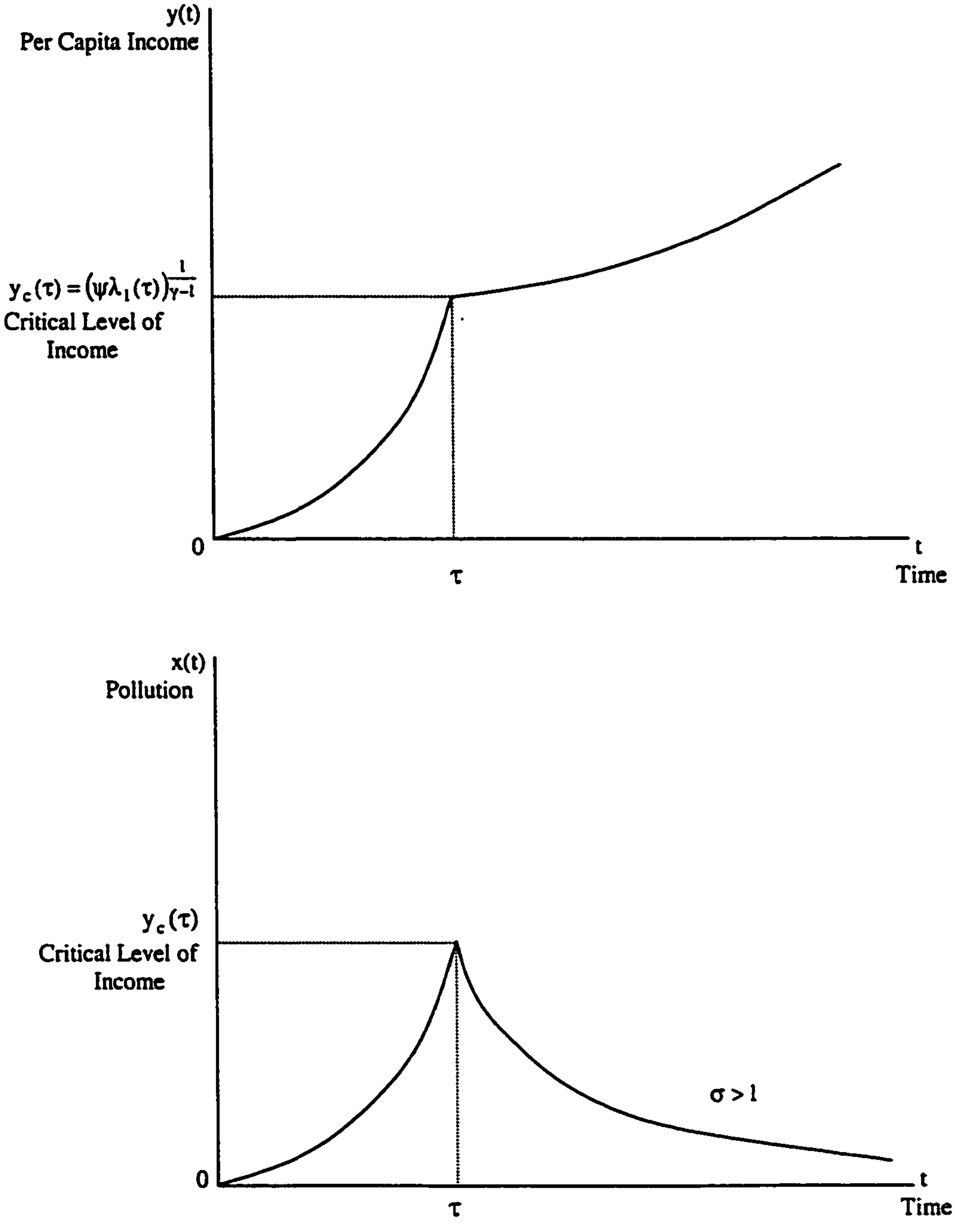

Figure 4.2: Optimal Time Paths of Per Capita Income and Pollution in the Case of $\sigma > 1$

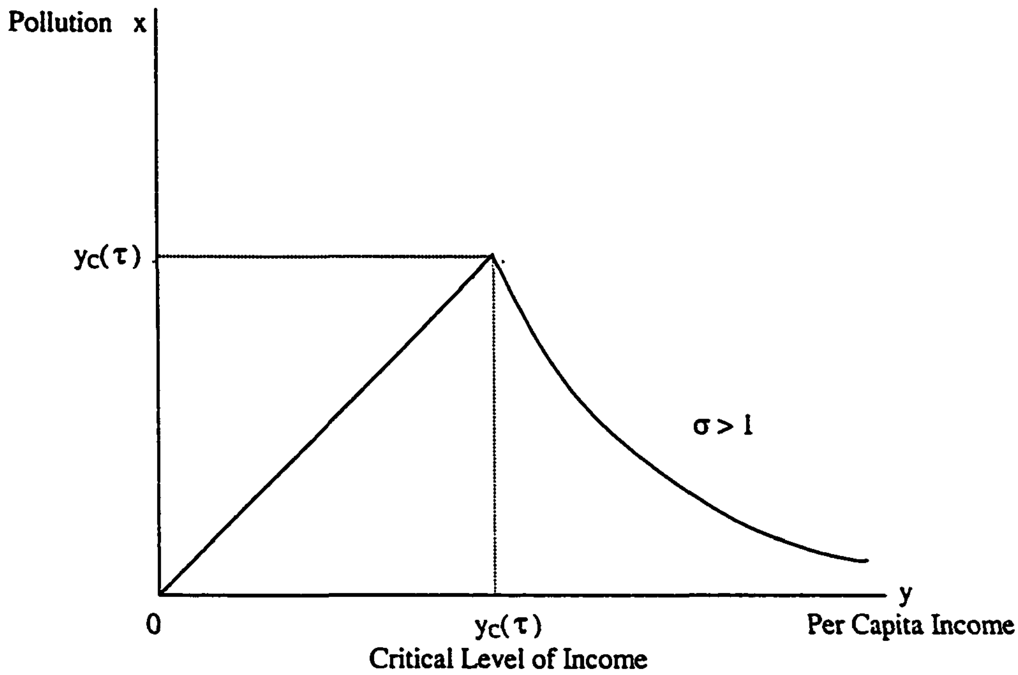

Figure 4.3: Relationship between Per Capita Income and Pollution in the Dynamic Model with $\sigma > 1$

Chapter 5

Long-Run Growth and Sustainable Development in the Presence of Pollution

In this chapter, we investigate long-run growth in the presence of pollution. By focusing on the long-run growth path, we deal with the central issue addressed in this literature—whether unbounded growth can be sustained with the maintenance of environmental quality. In particular, since one of the important objectives for the study of economic growth is to explore its implication on welfare, and environmental quality affects welfare in both direct and indirect ways, we use our model to study the issue of sustainable development. We interpret the term "sustainable development" in a specific way for the true welfare analysis, and we make a contribution to the literature of economic growth by applying the concept of sustainable development to the model of economic growth. Thus, our model can be used to compare with the welfare analysis of the conventional growth models that do not take into account the environmental externalities.

In order to discuss the economic effects of various types of pollution, we make a distinction between two cases in which pollution has its impact as a flow and as a stock, so both cases are studied in this chapter.

The rest of this chapter is structured as follows. Section 5.1 analyzes the basic dynamic model in which pollution is treated as a flow, the same one presented in the previous chapter in Section 4.2, but we focus on the (asymptotic) long-run growth path rather than the transition of the economy. We investigate the long-run growth implications and discuss the issue of sustainable development to compare with those of conventional endogenous growth models in which environmental concerns are ignored. Section 5.2 extends the basic model to examine the problem of pollution when it affects welfare through the cumulative stock effect.

5.1 Long-Run Growth and Sustainable Development with Optimal Control of Pollution Flow

5.1.1 Asymptotic Long-Run Growth Path

We analyze the simple two-sector endogenous growth model described in Chapter 3 to study long-run growth in the presence of pollution. In order to examine the possibility of long-run growth with optimal control of pollution, we consider the social planner's problem given in the previous chapter in (4.17).

In the long run the economy asymptotically approaches to the steady-state growth path[35] along which all endogenous variables grow at constant rates. In this section, we

[35] Since some authors in the literature of economic growth defined the term "balanced growth path" as a

focus on analyzing the asymptotic steady-state growth path rather than the transition, in order to investigate the long-run growth implications in the presence of pollution. Mathematical derivations for the optimality conditions and the asymptotic long-run growth rates of the endogenous variables in this model are provided in Appendix A. Let g_η denote the asymptotic long-run growth rate of η, where η = k, y, c, h, ω, and x. The common growth rate of physical capital, output, and consumption along the asymptotic steady-state growth path is

$$g_k = g_y = g_c = \frac{1}{\sigma+\vartheta}(\delta-\rho), \tag{5.1}$$

where $\vartheta \equiv \dfrac{\sigma+\gamma-1}{(1-\alpha)\beta\gamma} > 0$, and the growth rate of the human capital stock along the long-run steady-state growth path is

$$g_h = (1+\vartheta)g_y = \left(\frac{1+\vartheta}{\sigma+\vartheta}\right)(\delta-\rho) > g_y\,. \tag{5.2}$$

We assume that $\delta > \rho$, which is a necessary and sufficient condition for growth to be sustainable in a standard model of human capital with no consideration of environmental externalities. The assumption of $\delta > \rho$ is also necessary in this model for making the

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path along which all non-stationary variables are growing at a common, constant rate (e.g., Romer (1990), Victor, Chang and Blackburn (1994), Stokey (1998)), we use the more general term "steady-state growth path" to describe a path with constant (not necessarily the same and possibly zero) rates of production technology of human capital productive enough to avoid a corner solution with no human capital accumulation and no growth in both output and pollution. Because $\vartheta > 0$, human capital grows faster than physical capital, output, and consumption. If we do not take into account the environmental externalities in this model, output, consumption, physical capital, and human capital all would grow at the common rate of $\frac{1}{\sigma}(\delta - \rho)$. Hence, with the presence of environmental considerations, output, consumption, and physical capital stock grow slower than those in the model with no pollution. However, the comparison of the growth rate of human capital between two models depends on the value of σ. If $\sigma > 1$ ($1 > \sigma > 0$), human capital grows faster (slower) in the model with environmental considerations than in the model without pollution, and if $\sigma = 1$, the growth rates of human capital in the two models are the same.

We can also study the long-run behaviors of the optimal standard of environmental regulation (ω) and pollution. Recall that the level of pollution is

$$x(t) = \left(q(z(t))k(z(t))\right)^{\alpha}\left((1-u(t))\,h(t)\right)^{1-\alpha} = \left(\frac{1}{1+z(t)}\right)^{\alpha(\beta+1)} k(t)^{\alpha}\left((1-u(t))\,h(t)\right)^{1-\alpha}, \tag{5.3}$$

where $\beta > 1$, $z(t) \geq 0$, and $q(z(t)) = \left(\frac{1}{1+z(t)}\right)^{\beta} \equiv \omega(t)^{\beta}$ represents the pollution generating level per unit of type z(t) physical capital used in production at time t. To get

growth in the stock and control variables of the model.

the growth rates for ω and x, differentiate (4.21b) and (5.3) with respect to time and substitute the growth rates of consumption and physical and human capital stock given in (5.1) and (5.2) into them. When it is not optimal to control pollution in the early stage of economic development, ω is constant (i.e., $\omega=1$) and pollution grows faster than output but slower than human capital (i.e., pollution grows at the rate $g_x = (\alpha + (1-\alpha)(1+\vartheta))g_y$). Along the asymptotic long-run growth path after the potential output level exceeds the critical level, the quality of differentiated physical capital improves at the rate

$$g_\omega = \left(\frac{1-\gamma-\sigma}{\alpha\beta\gamma}\right)g_y = \left(\frac{1-\gamma-\sigma}{\alpha\beta\sigma}\right)\left(\frac{\delta-\rho}{\sigma+\vartheta}\right) < 0, \tag{5.4}$$

and the pollution level changes at the rate

$$g_x = \frac{1-\sigma}{\gamma}g_y = \left(\frac{1-\sigma}{\gamma}\right)\left(\frac{\delta-\rho}{\sigma+\vartheta}\right) < g_y. \tag{5.5}$$

As we have shown before, the growth rate of pollution along the asymptotic steady-state growth path critically depends on the value of σ. If $0<\sigma<1$, pollution grows at a constant (positive) rate but slower than output because $\gamma>1$. Pollution remains unchanged if $\sigma=1$, and declines if $\sigma>1$ along the asymptotic long-run growth path. Hence, if the condition of $\sigma>1$ is satisfied, this model yields the result that pollution follows an inverted U-shaped pattern over time, and that human capital grows faster in the presence of environmental considerations than in the absence of pollution. Moreover, if the condition of $\sigma > 1$ holds, it is sufficient for the transversality conditions for physical and human capital to be satisfied and the condition guarantees that the long-run growth path of the economy is at least locally stable around the steady-state growth path.[36]

Intuitively, the question of whether or not growth is sustainable is related to the behavior of the real rate of return on physical capital. Using the first-order condition with respect to consumption (5.2), the Euler equation for λ_1 (shadow value of physical capital) can be written as

$$\frac{\dot{c}}{c} = \frac{1}{\sigma}\left(\frac{\alpha\beta}{\beta+1}\frac{y}{k} - \rho\right) = \frac{1}{\sigma}\left[\left(\frac{\alpha\beta}{\beta+1}\right)\omega^{\alpha}\left(\frac{(1-u)h}{k}\right)^{1-\alpha} - \rho\right], \tag{5.6}$$

where the term $\frac{\alpha\beta}{\beta+1}\frac{y}{k}$ is the social marginal product of physical capital, which can be interpreted as the net gain in utility from an additional unit of physical capital. Measured in terms of marginal utility of consumption, the term represents the difference between additional utility from increased output for consumption and the decrease in utility due to increased pollution, both caused by an additional unit of physical capital. Hence, the real rate of return on physical capital is

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[36] The transversality conditions for physical and human capital hold if $(1-\sigma)g_k < \rho$, which is satisfied if $\sigma \geq 1$, or if $0<\sigma<1$ and $g_k < \rho/(1-\sigma)$. For derivation of the transversality conditions and the stability condition around the steady-atate growth path, see Appendix A and B.

$$r=\frac{\frac{\partial U}{\partial c}\frac{\partial y}{\partial k}-\frac{\partial U}{\partial X}\frac{\partial X}{\partial k}}{\frac{\partial U}{\partial c}}=\frac{\alpha\beta}{\beta+1}\frac{y}{k}=\left(\frac{\alpha\beta}{\beta+1}\right)\omega^{\alpha}\left(\frac{(1-u)h}{k}\right)^{1-\alpha}. \tag{5.7}$$

The essential feature to note in equation (5.10) is that k(t) and y(t) grow at the same rate along the asymptotic steady-state growth path so that the real rate of return to physical capital could remain constant indefinitely in the long run. This is only possible because human capital accumulates faster than physical capital by just enough to compensate for the fall in ω along the asymptotic long-run growth path, i.e., $g_h = g_k + \left(-\frac{\alpha}{1-\alpha}g_\omega\right)$. Thus, consumption in (5.9) can grow at a constant rate as long as the real rate of return to physical capital remains constant and is greater than the rate of time preference. This result of sustained growth in the presence of an environmental externality is contrasted with the *AK* model with pollution, developed by Stokey (1998). In her model, growth is not sustainable because the stricter emission standard reduces the real rate of return on physical capital below the rate needed for sustained growth. The major reason for the difference from Stokey's (1998) model in the sustainability of growth is that we make distinction between physical and human capital in terms of pollution generating levels in their own production processes. According to the *AK* approach in the presence of pollution, the optimal quality of differentiated physical capital improves as output level increases, which implies that we need more physical capital to produce the same unit of output as the cost of reducing pollution. Hence, the social marginal product of physical capital eventually declines below the rate of time preference in the presence of the optimal pollution control. In this model, by contrast, because the technology of producing human capital is clean and does not generate pollution, the presence of pollution does not reduce the social marginal product of human capital. Furthermore, the social marginal product of physical capital can remain constant as long as human capital grows faster than physical capital, and it has a positive effect on the marginal product of physical capital to offset the cost of using more physical capital for pollution control.

5.1.2 Sustainable Development

The term "sustainable development" was first introduced by the World Commission on the Environment and Development (1987) (the 'Brundtland' Commission) and was defined as "development that meets the needs of present generation without compromising the ability of future generations to meet their needs".

Economic growth that leads to ultimate environmental degradation might make us worse off. So we can interpret the "needs" in the above definition of sustainable development as the basic demand of each generation for both material consumption goods and the environmental quality that provides economic service as a public consumption good. Thus, sustainable development could be interpreted in general as development that takes into account the welfare of future generations as well as that of present generation, which depend not only on the consumption of produced goods but also on the environmental quality. Since pollution directly affects our welfare by harming human health and/or by damaging the amenity value of the environment, the instantaneous utility function as a true measure of welfare should be defined over pollution as well as consumption of produced goods. Likewise, the green concept of income developed by Hartwick (1990) and Bovenberg and Smulders (1994)[37], which accounts for the imputed income associated with environmental amenities in addition to other conventional components of income, could be a true measure of permanent income.

Although there is no working definition of sustainable development to use in an analytical framework, we follow Byrne (1997) and Aghion and Howitt (1998) in the sense that the long-run growth of utility could be an appropriate measure of sustainable development. Therefore, in this section, we analyze the long-run growth of instantaneous utility as an index of sustainable development, which incorporates the growth rates of both consumption and pollution. Differentiating the instantaneous utility function U=U(c,x) with respect to time yields the growth of instantaneous utility over time as

$$\dot{U} = U_c \dot{c} + U_x \dot{x}, \text{ or } \frac{\dot{U}}{U} = \left(\frac{U_c c}{U}\right)\frac{\dot{c}}{c} + \left(\frac{U_x x}{U}\right)\frac{\dot{x}}{x}. \tag{5.8}$$

Because $U_c > 0$ and $U_x < 0$, instantaneous utility grows over time if $\dot{c} > 0$ and $\dot{x} < 0$ ($\sigma > 1$), or if U_x is not too large to be greater than $U_c \dot{c}/\dot{x}$ in the absolute values when

[37] Hartwick (1990) takes into account the stock of natural resources (both exhaustible and renewable) and pollution to obtain a measure of true net national product (NNP). Bovenberg and Smulders (1994) developed the concept of green income by incorporating the imputed income from consumption of environmental amenities and investment in environmental quality into the components of conventional income.

$\dot{x} > 0$ ($\sigma < 1$), which implies that $U > 0$. Along the asymptotic long-run growth path, the instantaneous utility grows at the rate

$$g_U = \left(\frac{U_c c}{U}\right) g_y + \left(\frac{U_x x}{U}\right)\left(\frac{1-\sigma}{\gamma}\right) g_y = (1-\sigma) g_y . \qquad (5.9)$$

Hence, the social utility grows at a constant rate along the asymptotic long-run growth path except when $\sigma = 1$. If $\sigma = 1$ for the case of logarithmic utility function of consumption, the asymptotic growth rate of the instantaneous utility is zero. As the growth rate of output in the absence of environmental considerations is greater than that in the presence of pollution, the same holds for the utility growth rate.[38] Although the pollution level increases if $0 < \sigma < 1$, or declines if $\sigma > 1$, along the asymptotic long-run growth path, the social utility increases in both cases. In case of $0 < \sigma < 1$, g_U is positive so that $\dot{U} > 0$ and $U > 0$. In case of $\sigma > 1$, both $U < 0$ and $g_U < 0$ hold, implying that $\dot{U} > 0$. Hence, the instantaneous utility as a measure of the standard of living affected by environmental quality as well as consumption improves over time, and consumers become better off along the asymptotic long-run growth path. Moreover, as σ becomes larger, pollution declines faster and social utility improves at a higher rate. Hence, utility growth can be sustained regardless of the magnitude of σ as long as $\sigma \neq 1$. Sustained growth of social utility or sustainable development is possible due to the fact that when

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[38] The growth rate of utility in the absence of pollution is $(1-\sigma)g_y^n$, where g_y^n is the growth rate of output in a standard human capital model with no pollution.

pollution is linked with output growth, the social planner chooses the cleaner production technology by allocating resources to internalize the negative externalities. When pollution does affect utility, the utility growth in a decentralized economy cannot be the same as that obtained by the social planner's optimal solutions, which we will also investigate later. Some previous studies in this literature often argue that a possible path for sustainable development with the maintenance of environmental quality is no economic growth. This model shows that the growth of social utility as an index of sustainable development can be sustained with an optimal control of pollution when the important sources of growth are not affected by the presence of pollution.

5.2 Pollution Stock

So far, we have treated pollution as a flow. Some types of pollution dissipate very rapidly due to the natural assimilative and regenerative capacity of the environment, if there were not continuous inflows of new pollutants (e.g., noise, sulfur dioxide, particulates, and certain types of organic water pollution). In this case, there is no stock accumulation effect for pollution, and it is reasonable to assume that pollution affects utility through the flow effect. However, some other types of pollution (environmental degradation) are cumulative, and they reduce the regenerative and pollution absorbing capacity of the environment. Therefore, the quality of the natural environment is more slowly recovered than the previous case (e.g., agricultural soil quality, radioactive materials, depletion of the ozone layer, deforestation, water quality and the fish stock, and fisheries). In this case, it is more reasonable to assume that pollution affects utility through the cumulative stock effect.

For an extensive analysis of the long-run behaviors of the economy in the presence of pollution, we will also consider the case in which pollution has its impact as a stock in this section. By developing a new model in which pollution is treated as a stock, we examine if this approach yields different results and implication from those obtained in the model where pollution enters utility as a flow.

5.2.1 The Model

Assume that the pollution stock, X(t), accumulates by the gross inflow of new pollution generated in the process of final output production, $x(t) = \omega^{\alpha(\beta+1)}(t)k^{\alpha}(t)((1-u(t))h(t))^{1-\alpha}$, which was the pollution flow that affects utility in the previous model. Also, the stock of pollution is assumed to decay at a fixed rate, $\varepsilon \geq 0$, to reflect the natural assimilative or regenerative capacity of the environment. So, the net of pollution stock accumulates over time according to the following equation:

$$\dot{X}(t) = \omega^{\alpha(\beta+1)}(t)\, k^{\alpha}(t)((1-u(t))h(t))^{1-\alpha} - \varepsilon X(t), \tag{5.10}$$

where $\varepsilon \geq 0$ is the natural decay rate of the pollution stock, which is assumed to be fixed, and all other notations are the same as before. Therefore, when pollution accumulates as a stock that affects utility, the social planner's problem can be written as

$$\max \int_0^\infty e^{-\rho t} \left[\frac{c^{1-\sigma}(t)}{1-\sigma} - \frac{\phi X(t)^\gamma}{\gamma} \right] dt \tag{5.11}$$

$$\text{s.t.} \quad \dot{k}(t) = \omega^\alpha(t) k^\alpha(t) \left((1-u(t)) h(t) \right)^{1-\alpha} - c(t),$$

$$\dot{h}(t) = \delta u(t) h(t),$$

$$\dot{X}(t) = \omega^{\alpha(\beta+1)}(t)\, k^\alpha(t) \left((1-u(t)) h(t) \right)^{1-\alpha} - \varepsilon X(t),$$

$$\omega(t) = \frac{1}{1+z(t)}, \; z(t) \geq 0, \; \omega(t) \leq 1,$$

where all notations are the same as those in the previous section except pollution, and the initial stocks of physical and human capital, and pollution are given as $k(0)=k_0$, $h(0)=h_0$, and $X(0)=X_0$, respectively. The current value Hamiltonian of this problem is given as

$$\begin{aligned} H = \frac{c^{1-\sigma}}{1-\sigma} - \frac{\phi X^\gamma}{\gamma} + \lambda_1 \left[\omega^\alpha k^\alpha \left((1-u) h \right)^{1-\alpha} - c \right] + \lambda_2 \delta u h \\ - \lambda_3 \left[\omega^{\alpha(\beta+1)} k^\alpha \left((1-u) h \right)^{1-\alpha} - \varepsilon X \right] + \mu (1-\omega), \end{aligned} \tag{5.12}$$

where λ_1, λ_2, and λ_3 denote the costate variables associated with k, h, and X, respectively. Since the shadow value of pollution stock represents the marginal damage caused by a unit increase in pollution stock, the sign of the costate variable for X is reversed so that we could measure λ_3 in the absolute value, i.e., $\lambda_3 > 0$. As before, μ is a Lagrange multiplier associated with the inequality constraint for the quality index of the differentiated physical capital used in production, $\omega \equiv \frac{1}{1+z}$; i.e., $z \geq 0 \Rightarrow \omega \leq 1$.

5.2.2 Transition

The dynamics of this model are more complicated than those of the previous one in which pollution is treated as a flow, because we have now three state variables (k, h, and X) as well as three choice variables (c, u, and ω). Appendix C contains the mathematical derivations of the optimality conditions and the asymptotic steady-state growth path for this model. Along the optimal path, consumption and the quality of differentiated physical capital should satisfy

$$c^{-\sigma} = \lambda_1 \tag{5.13}$$

$$\omega = \begin{cases} 1, & \text{if } \lambda_1 \geq \lambda_3(\beta+1), \\ \left(\dfrac{\lambda_1}{\lambda_3(\beta+1)}\right)^{\frac{1}{\alpha\beta}}, & \text{if } \lambda_1 < \lambda_3(\beta+1). \end{cases} \tag{5.14}$$

From (5.17), we see that the economy in this model also has a transition. Assume that the stocks of physical capital and pollution are both small at the initial stage of economic growth, then the shadow price of physical capital, λ_1, could be sufficiently high relative to that of the pollution stock, λ_3, so that $\lambda_1 \geq \lambda_3(\beta+1)$. In this case, only the dirtiest physical capital is used in the final output production (i.e., $\omega = 1$). When pollution is not controlled at all, the gross inflow of new pollution generated in the process of final output production increases at the same rate as that of the final output. Therefore, both the inflow of new pollution and the pollution stock rise in the region where $\omega = 1$. Over time, however, the shadow value of physical capital (λ_1) falls, and the shadow value of pollution stock (λ_3) rises as the stocks of both physical capital and pollution grow. Since the shadow value of physical capital relative to that of pollution stock, λ_1 / λ_3, declines monotonically along the transition path, there is a critical point in time, defined by τ such that $\lambda_1(\tau) = \lambda_3(\tau)(\beta+1)$. Thereafter, it becomes optimal to control pollution by using the higher quality (i.e., cleaner) physical capital in the final output production (i.e., $\omega < 1$), and furthermore, ω declines over time, which implies that pollution is more strictly controlled as the economy grows. As the economy asymptotically approaches the steady-state growth path after passing the transition path, the quality of differentiated physical capital, ω, as an index of clean technology, improves at the rate

$$\frac{\dot{\omega}}{\omega} = \frac{1}{\alpha\beta}\left(\frac{\dot{\lambda}_1}{\lambda_1} - \frac{\dot{\lambda}_3}{\lambda_3}\right) < 0. \tag{5.15}$$

Hence, the dynamic behavior of the optimal choice of differentiated physical capital in this model exhibits the same pattern as that in the previous model of pollution flow. However, the decision for both timing and strictness of pollution control in this model depends on the shadow values of both physical capital and pollution stock, while the shadow value of physical capital affected the decision for pollution control in the previous model.

5.2.3 Asymptotic Long-Run Growth Path

We are now in a position to investigate the long-run growth implications when pollution has its impact as a stock. We focus on the steady-state growth path, which we define as a path along which all the variables grow at constant rates, although not necessarily the same rates, while the fraction of non-leisure time allocated to human capital accumulation is constant. The asymptotic long-run growth rates of the interested variables are

$$g_y = g_k = g_c = \frac{1}{\sigma + \vartheta}(\delta - \rho) \text{ where } \vartheta = \frac{\sigma + \gamma - 1}{(1-\alpha)\beta\gamma} > 0, \tag{5.16}$$

$$g_h = (1 + \vartheta)g_y > g_y, \tag{5.17}$$

$$g_{\omega} = \left(\frac{1-\gamma-\sigma}{\alpha\beta\gamma}\right) g_y < 0, \tag{5.18}$$

$$g_X = \frac{1-\sigma}{\gamma} g_y, \tag{5.19}$$

The asymptotic long-run growth rates of consumption, output, physical and human capital, and the quality of differentiated physical capital for this model are exactly same as those for the previous model in which pollution enters utility as a flow. Furthermore, the growth rate of pollution stock along the asymptotic steady-state growth path is the same as that of pollution flow in the previous model, and it is independent of the natural decay rate, ε. The underlying implication behind these results can be derived directly from the law of motion given in equation (5.10). If we divide both sides of equation (5.10) by the pollution stock, X(t), then we get

$$\frac{\dot{X}(t)}{X(t)} = \frac{x(t)}{X(t)} - \varepsilon, \tag{5.20}$$

where x(t) denotes the gross inflow of new pollution at time t. If the pollution stock grows at a constant rate along the steady-state growth path, then $\frac{x(t)}{X(t)}$ must be also constant in equation (5.23), which implies that the growth rate of pollution stock is the same as that of the inflow of new pollution (i.e., pollution flow in the previous model).

Hence, the dynamic behavior of pollution stock along the transition path in this model displays the same pattern as that of pollution flow in the previous model. If $\sigma > 1$ holds, the pollution stock increases with no pollution control in the early stage of economic growth, but as the economy asymptotically approaches the steady-state growth path, the pollution stock decreases at the rate $\frac{1-\sigma}{\gamma} g_y < 0$. Therefore, the pollution stock displays an inverted U-shaped pattern over time as long as $\sigma > 1$ holds.

Note that there is also a transversality condition for pollution stock in this model. The transversality conditions are satisfied if $(1-\sigma)g_y < \rho$, which is the same as before. In this model, however, there is another inequality for the existence of the optimal solution. From the law of motion of X given in (5.10), we get

$$\frac{\omega^{\alpha\beta} y}{X} = g_X + \varepsilon = \frac{(1-\sigma)g_y}{\gamma} + \varepsilon > 0 \Rightarrow (1-\sigma)g_y > -\varepsilon\gamma . \tag{5.21}$$

Combining the transversality conditions with (5.21), we see that the optimal solution of this model exists if and only if

$$-\varepsilon\gamma < (1-\sigma)g_y < \rho . \tag{5.22}$$

Hence, if σ is greater than one (i.e., elasticity of the marginal utility of consumption is greater than one), the natural decay rate of pollution stock, ε, should be large enough for the existence of a solution in this model. For example, if $\sigma > 1$ and $\varepsilon = 0$ (i.e., the environment does not have its own self-correcting nature for the environmental degradation), then the optimal solution of this model may not exist.

Chapter 6

Growth and Pollution in a Decentralized Economy and Policy Analysis for the Social Optimum

So far, we have studied the issue of economic growth and pollution in the context of social planner's problem. We characterized the optimal dynamic behaviors of consumption, saving, the allocation of non-leisure time between two sectors, and the pollution level along both transitional path and the asymptotic long-run growth path.

However, without government intervention, the decentralized economy generally suffers from a market failure associated with the negative externality of pollution. Although consumers benefit from better environmental quality, there is generally no market for environmental quality or pollution. Consumers take the pollution level as given while they make decisions on consumption, saving, and the allocation of their labor between additions to human capital and final goods production. Producers, on the other hand, would face no cost at all for pollution, but only a benefit from producing more output with more pollution input. In the absence of government regulation of pollution, a cleaner technology can be chosen at the expense of fewer goods produced. Therefore, the producers will always choose the dirtiest technology to maximize their output and profit.

In this chapter we study the equilibrium growth paths of a decentralized economy with and without government intervention. First, without government intervention, we compare the equilibrium solutions with the optimal ones that have been found by solving the social planner's problem. Second, we examine the possibility of whether or not sustainable development can be achieved in a decentralized economy without government intervention. Last, we introduce the government's roles in order to study the issue of implementing the social optimum in a decentralized economy. In particular, two specific policy tools—pollution tax and pollution voucher (permit)—are analyzed in detail to see how they work in a market mechanism. We examine whether any of these policy instruments might implement the optimal sustainable growth paths in the long run in a decentralized economy.

This chapter uses the same model as that described in Chapter 3, but the economy is decentralized into the household's and firm's problems to solve for a competitive equilibrium in a decentralized economy. Also, the pollution is treated as a normal input in the firm's problem to analyze the firm's behavior on the input mix between pollution and other conventional inputs.

The remainder of this chapter is organized as follows. A description of the model of growth and pollution in a decentralized economy is provided in section 6.1. We analyze the household and firm's problems and study the equilibrium paths without government intervention. Section 6.2 investigates the possibility of sustainable development in a decentralized economy without government. In section 6.3 we study the issue of implementing the social optimum in a decentralized economy with two specific kinds of policy instruments.

6.1 Growth and Pollution in a Decentralized Economy without Government Intervention

We assume that there is an infinitely lived representative household and a representative firm. Both act as price takers in all markets.

6.1.1 The Household's Problem

The representative household owns physical and human capital. The household takes as given the rates of return on physical and human capital, r(t), w(t)[39], respectively.

As before, physical capital is differentiated into infinitely many physical capital goods with respect to cost and cleanliness. Our model implicitly assumes that general physical capital can be used for either production or pollution abatement. The pollution level depends on the firm's production technique in terms of how it uses general physical capital. Therefore, a quality index of differentiated physical capital ($\omega \equiv \frac{1}{1+z}, z \geq 0$) can

[39] If we interpret human capital as effective labor, then w(t) is the wage of effective labor that is defined as $h(t) = A(t)\,\ell(t)$, where A(t) is effectiveness of each worker and $\ell(t)$ is the raw labor. If $w_r(t)$ is the wage for the raw labor, then $w(t) = w_r(t)/A(t)$. In our model, the wage income per household is w(t)h(t).

be understood as a portion of general physical capital devoted to production, while the remaining $(1-\omega)$ is devoted to pollution abatement. Depending on the existence and strictness of government regulation on pollution, firms decide which type of differentiated physical capital to use in production. On the other hand, the individual household owns general physical capital, k(t), and receives rental income, r(t)k(t), by renting it to firms. Given the household's income, general physical capital accumulates by the household's saving decision.

Also, each household owns one unit of non-leisure time per period. The household with human capital, h(t), devotes the fraction, 1-u(t), of non-leisure time to production, and receives wage income, w(t)(1-u(t))h(t). The household devotes the remaining, u(t), of non-leisure time to human capital accumulation. The basic incentive of human capital accumulation is the same as that of saving, i.e., to increase the lifetime income and consumption. The opportunity cost of human capital accumulation at time t is the forgone wage by not working, which equals w(t)u(t)h(t). If all the non-leisure time is devoted to working, the household's wage income is w(t)h(t). Our model implicitly assumes that the household's potential wage income is distributed to consumption, c(t), and saving, $\dot{k}(t)$, by the amount of w(t)(1-u(t))h(t), and to investment in human capital, $\dot{h}(t)$, by the amount of w(t)u(t)h(t). Consequently, the allocation decision of non-leisure time between production and human capital accumulation is the same as the distribution decision of potential wage income to consumption and saving, and to investment in human capital. Hence, the household will choose the fraction, u(t), of non-leisure time in a way that the forgone wage should equal to the implicit value of additional human capital produced by the human capital accumulation equation, $\dot{h}(t) = \delta u(t)h(t)$, where $\delta > 0$ is the productivity parameter.

We assume that the individual household takes the pollution level, x(t), as given in a decentralized economy. In fact, the pollution level, x(t), varies over time by the firm's decision of how much to pollute even if the household takes x(t) as given.

Taking as given the rates of return on physical and human capital and the pollution level, the household will choose the time paths for consumption, c(t), saving, $\dot{k}(t)$, and the fraction, u(t), of non-leisure time devoted to human capital accumulation to maximize the lifetime utility. So the infinitely lived representative household's problem is

$$\max \int_0^\infty e^{-\rho t}\left(\frac{c(t)^{1-\sigma}}{1-\sigma} - \frac{\phi x(t)^\gamma}{\gamma}\right)dt \tag{6.1}$$

$$\text{s.t.} \quad \dot{k}(t) = r(t)k(t) + w(t)(1 - u(t))h(t) - c(t),$$

$$\dot{h}(t) = \delta u(t)h(t),$$

where all notations are the same as before and the initial stocks of physical and human capital are given as $k(0) = k_0 > 0$ and $h(0) = h_0 > 0$, respectively, and r(t), w(t), and x(t) are taken as given. The current value Hamiltonian for the representative household's problem is

$$H = \frac{c^{1-\sigma}}{1-\sigma} - \frac{\phi x^{\gamma}}{\gamma} + \lambda_1\big(rk + w(1-u)h - c\big) + \lambda_2 \delta uh, \tag{6.2}$$

where λ_1 and λ_2 denote the costate variables associated with physical and human capital, respectively.

The first-order conditions with respect to c and u are

$$\frac{\partial H}{\partial c} = 0 \Rightarrow c^{-\sigma} = \lambda_1 \Rightarrow \frac{\dot{c}}{c} = -\frac{1}{\sigma}\frac{\dot{\lambda}_1}{\lambda_1}, \tag{6.3}$$

$$\frac{\partial H}{\partial u} = 0 \Rightarrow \lambda_1 w = \lambda_2 \delta \text{ or } w = \frac{\lambda_2}{\lambda_1}\delta. \tag{6.4}$$

In equation (6.4), w is the return on human capital, which should be equal to the marginal product of human capital employed in the final output production sector in equilibrium. On the other hand, $\frac{\lambda_2}{\lambda_1}$ is the shadow price of human capital relative to physical capital, and δ is the marginal product of human capital employed in the human capital investment sector. Hence the equation (6.4) represents the condition for optimal static allocation of human capital. Given the total amount of human capital at any point in time, human capital should be allocated so that the marginal product of human capital, measured in terms of units of physical capital, is the same between two sectors.

Furthermore, if we substitute (6.4) into the human capital accumulation equation given in (6.1), we get

$$\left(\frac{\lambda_2(t)}{\lambda_1(t)}\right)\dot{h}(t) = w(t)u(t)h(t). \tag{6.5}$$

The left-hand-side of equation (6.5) is the value of new human capital measured in units of physical capital, and the right-hand-side of equation (6.5) represents the forgone wage by not working the fraction, u(t), of non-leisure time. Therefore, equation (6.5) indicates that the household spends a part of potential wage income, by the amount of w(t)u(t)h(t), for additions to human capital.

The Euler equations for λ_1 and λ_2 are

$$\dot{\lambda}_1 = \rho\lambda_1 - \frac{\partial H}{\partial k} \Rightarrow \frac{\dot{\lambda}_1}{\lambda_1} = \rho - r(t), \tag{6.6}$$

$$\dot{\lambda}_2 = \rho\lambda_2 - \frac{\partial H}{\partial h} \Rightarrow \dot{\lambda}_2 = \rho\lambda_2 - \left(\lambda_1 w(1-u) + \lambda_2\delta u\right). \tag{6.7}$$

Using equation (6.4), equation (6.7) simplifies to

$$\frac{\dot{\lambda}_2}{\lambda_2} = \rho - \delta. \tag{6.8}$$

By combining equations (6.6) and (6.8), or (6.3) and (6.6), the rate of return on physical capital r(t) can be expressed, respectively, as

$$r(t) = \delta + \frac{\dot{\lambda}_2}{\lambda_2} - \frac{\dot{\lambda}_1}{\lambda_1} = \delta + \left(\frac{\dot{\lambda}_2}{\lambda_1}\right) \Big/ \left(\frac{\lambda_2}{\lambda_1}\right), \text{or} \tag{6.9a}$$

$$r(t) = \rho - \frac{\dot{U}_c}{U_c}. \tag{6.9b}$$

Equations (6.9a) and (6.9b) represent the conditions for optimal dynamic allocation, which indicate that the return from investment in the two different kinds of capital (i.e., k and h) should be traded off against each other and against consumption. More specifically, equation (6.9a) represents an arbitrage condition that physical and human capital investment should yield the same return. That is, the return on physical capital, which equals the marginal product of physical capital, should be the same as the return on human capital. As for the return on human capital, we should note that it accounts for not only the marginal product of human capital employed in the human capital investment, but also the change in the price of human capital relative to physical capital. On the other hand, equation (6.9b) represents the Keynes-Ramsey rule accounting for the trade-off between investment and consumption. Since the investment (in new physical capital) is accompanied by giving up the current consumption, the rate of return on investment should be equal to the rate of return on consumption, which is the rate of time preference and the rate of change in marginal utility of consumption. Since $\dot{U}_c = U_{cc}\dot{c}$, the second equation also implies that consumption will increase (decrease) over time, as the rate of interest is greater (smaller) than the rate of time preference, respectively.

6.1.2 The Firm's Problem

As we can see in equations (6.3), (6.4), and (6.6) in the household's problem, the household's decisions about consumption and the allocation of human capital between two sectors are determined by the rates of return on physical and human capital, r(t), w(t), respectively.

Now we consider the firm's problem to determine the competitive equilibrium prices of physical and human capital, r(t) and w(t). The representative firm's problem is to maximize profit at each point of time t, taking the rental rates of physical and human capital, r(t), w(t), as given. The firm is only interested in the amount of human capital to hire, so let $h_y(t)$ denote the human capital employed in the output production, i.e., $h_y(t) \equiv (1-u(t))h(t)$. The firm makes decisions on how much physical and human capital to employ, taking their rental rates as given. Furthermore, the firm makes a decision about what technology to use, in terms of emission rate, by making a choice of the quality of differentiated physical capital to be used in production (i.e., $\omega(t)$). The firm's choice of technology faces a trade-off between productivity and environmental quality. The use of higher quality physical capital is clean but less productive, and the lower quality physical capital is dirty but produces more output. As a consequence, the firm's choice of technology, $\omega(t)$, is the same as choosing how much pollution, x(t), to generate.

Therefore, the representative firm's problem is the static one of choosing its input levels, k(t), $h_{y(t)}$, and the quality of differentiated physical capital, $\omega(t)$, as an index of clean technology, to maximize profit at any given point of time t as below:

$$\max_{\omega,k,h_y} \pi = (\omega k)^{\alpha} h_y^{1-\alpha} - (rk + wh_y) \tag{6.10}$$

s.t. $\omega \leq 1$.

Without government intervention, producers face no cost at all for generating pollution, but only a benefit from producing more output. Thus, producers have no incentive to abate pollution at the expense of fewer goods produced, which implies that the firm always uses the dirtiest physical capital, so $\omega(t) = 1$[40] at any time t.

Hence, we can rewrite the firm's problem as an unconstrained maximization problem as

[40] The Lagrangian for this problem is $L = \omega^{\alpha} k^{\alpha} h_y^{1-\alpha} - rk - wh_y + \mu(1-\omega)$, where $\mu \geq 0$ is the Lagrange multiplier associated with the inequality constraint for the quality index of differentiated physical capital ω. Applying the Kuhn-Tucker condition, the first-order necessary condition for a constrained maximum with respect to ω is $\frac{\partial L}{\partial \omega} = 0$, $\mu \geq 0$, $\mu(1-\omega) = 0 \Rightarrow \mu = \alpha\omega^{\alpha-1} k^{\alpha} h_y^{1-\alpha} > 0 \Rightarrow \omega = 1$.

$$\max \pi = k^{\alpha} h_y^{1-\alpha} - rk - wh_y \ . \tag{6.11}$$

The first-order conditions for a maximum with respect to k and h_y are

$$\frac{\partial \pi}{\partial k} = 0 \Rightarrow r = \alpha \left(\frac{h_y}{k} \right)^{1-\alpha}, \tag{6.12}$$

$$\frac{\partial \pi}{\partial h_y} = 0 \Rightarrow w = (1-\alpha) \left(\frac{k}{h_y} \right)^{\alpha}. \tag{6.13}$$

Hence the competitive equilibrium prices of physical and human capital, $r^*(t)$ and $w^*(t)$, are determined by the first-order conditions for the firm's problem given in (6.12) and (6.13). Given $r^*(t)$ and $w^*(t)$, the household's decisions on consumption and the allocation of human capital between two sectors solve the household's problem. Because the production technology exhibits constant returns to scale in k and h_y, the equilibrium prices of physical and human capital determined in (6.12) and (6.13) also satisfy the firm's zero-profit condition:

$$y = k^{\alpha} h_y^{1-\alpha} = rk + wh_y . \tag{6.14}$$

6.1.3 Balanced Equilibrium Growth Path

Now we combine the household's and firm's problems to solve for the competitive equilibrium in a decentralized economy without government intervention. The first goal of our research in this chapter is to study the equilibrium growth paths for consumption, output, physical and human capital, and the pollution level in order to compare with the optimal ones that have been found in the social planner's problem.

Substituting the equilibrium price of human capital given in (6.13) into (6.4), we can rewrite equation (6.4) as

$$\frac{\partial H}{\partial u} = 0 \Rightarrow \lambda_1 (1-\alpha)\frac{y}{1-u} = \lambda_2 \delta h. \tag{6.15}$$

Substituting the equilibrium price of physical capital given in (6.12) into (6.6), we get

$$\dot{\lambda}_1 = \rho\lambda_1 - \frac{\partial H}{\partial k} \Rightarrow \dot{\lambda}_1 = \rho\lambda_1 - \alpha\lambda_1 \frac{y}{k} \Rightarrow \frac{\dot{\lambda}_1}{\lambda_1} = \rho - \alpha\frac{y}{k}. \tag{6.16}$$

In addition to the first-order conditions, we also have the laws of motion of k and h, the final output production function, and the transversality conditions for the equilibrium solutions.

By combining the household's budget constraint given in (6.1) and the firm's zero-profit condition given in (6.14), the law of motion of k is given as

$$\dot{k} = y - c \Rightarrow \frac{\dot{k}}{k} = \frac{y}{k} - \frac{c}{k}. \tag{6.17}$$

The law of motion of h is

$$\dot{h} = \delta uh \Rightarrow \frac{\dot{h}}{h} = \delta u. \tag{6.18}$$

The production function of final output is given as

$$y = k^{\alpha}\left((1-u)h\right)^{1-\alpha} \Rightarrow \frac{\dot{y}}{y} = \alpha\frac{\dot{k}}{k} + (1-\alpha)\left(\frac{\dot{h}}{h} - \left(\frac{u}{1-u}\right)\frac{\dot{u}}{u}\right). \tag{6.19}$$

Now we focus on the steady-state growth path along which all the above conditions are satisfied and the variables c, y, k and h grow at constant rates while u is constant, so $\frac{\dot{u}}{u} = 0$. Let g_{ℓ} denote the long-run (steady-state) growth rate of the interested variable ℓ (ℓ=c, y, k, h, and etc.).

From equation (6.16), if we assume that λ_1 grows at a constant rate along the steady-state growth path, then y/k must be constant, implying that k and y grow at the same rate. Likewise, from the law of motion of k given in (6.17), c/k should be constant along the steady-state growth path, which implies that c and k grow at the same rate.

Also from the production function given in (6.19), we see that the growth rate of h is the same as the growth rates of k and y so that

$$\frac{\dot{k}}{k}=\frac{\dot{y}}{y}=\frac{\dot{c}}{c}=\frac{\dot{h}}{h}\Rightarrow g_k=g_y=g_c=g_h. \tag{6.20}$$

Unlike the social planner's problem in which h grows faster than k, y, and c, they all grow at the same rate along the steady-state growth path in a decentralized economy without government intervention.

Taking logarithms and differentiating equation (6.15) with respect to time, and combining equations (6.3), (6.8) and (6.20), we arrive at

$$\begin{aligned} &g_k=g_y=g_c=g_h=\frac{1}{\sigma}(\delta-\rho), \text{ and} \\ &u^*=\frac{1}{\delta\sigma}(\delta-\rho). \end{aligned} \tag{6.21}$$

We assume $\delta>\rho$ so that the growth rates of k, y, c, and h, and the equilibrium value of non-leisure time devoted to human capital accumulation, u^*, are positive. If $\delta\le\rho$, then there is no growth in k, y, c, and h, and there is no human capital accumulation, which is not the intent of this research. Also, u^* should be less than one in order to satisfy the transversality conditions, so $0 < u^* < 1$. The transversality conditions are

$$\lim_{t\to\infty} e^{-\rho t}\lambda_1(t)k(t) = 0 \Rightarrow \lim_{t\to\infty} e^{((1-\sigma)g_k-\rho)t}c(0)^{-\sigma}k(0) = 0, \tag{6.22}$$

$$\lim_{t\to\infty} e^{-\rho t}\lambda_2(t)h(t) = 0 \Rightarrow \lim_{t\to\infty} e^{-\rho t}\frac{1}{\delta}\left(\frac{1-\alpha}{1-u}\right)\lambda_1(0)y(0) = 0. \tag{6.23}$$

Thus, the above transversality conditions hold if $(1-\sigma)g_k < \rho$, which is satisfied if

$$\rho > (1-\sigma)\delta \Rightarrow u^* = \frac{1}{\delta\sigma}(\delta-\sigma) > 0. \tag{6.24}$$

While the growth rates of y, k and c are higher in a decentralized economy with no government intervention than those in the social planner's problem, the growth rate of human capital h is higher in the social planner's problem than that in a decentralized economy. These equilibrium solutions are different from the optimal solutions because consumers treat pollution as given and producers have no incentive to abate pollution.

However, the pollution level is, in effect, determined by the one-to-one relationship between output and pollution[41], and accordingly, the pollution level increases at the same rate as output growth rate as below:

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[41] In our model, pollution does not exceed the maximum level that is determined by the output capacity of the economy even if there is no government intervention in a decentralized economy. This is different from the models of Tahvonen and Kuuluvainen (1993), López (1994), and Bovenberg and Smulders (1995). Since they treat pollution as a normal input that can be used free of charge, producers can select an infinitely high level of pollution until the marginal cost of pollution is zero if there is no government intervention in a market economy.

$$x(t) = k(t)^{\alpha}\left((1-u(t))h(t)\right)^{1-\alpha} = y(t) \Rightarrow g_x = g_y. \tag{6.25}$$

In comparison with the optimal pollution control model in which the asymptotic long-run growth rate of pollution can be positive or negative depending on the value of σ, pollution grows at the same rate as output in a decentralized economy without government intervention. Moreover, while the economy asymptotically approaches the long-run growth path after passing its transition from $\omega = 1$ to $\omega < 1$ in the social planner's problem, the decentralized economy has no transition because $\omega = 1$ all the time. Thus, pollution cannot display the inverted U-shaped pattern in a decentralized economy if there is no government intervention. These results also come from the fact that households do not take into account the negative externality of pollution and that producers do not try to abate pollution, thus always choosing the dirtiest manufacturing process.

6.2 Sustainable Development in a Decentralized Economy

The second goal of our research in this chapter is to examine whether or not sustainable development can be achieved in a decentralized economy without government intervention. If not, we will also examine the possibility of sustainable development in a decentralized economy with government intervention by studying the issue of implementing the social optimum.

As before, we analyze the long-run growth rate of instantaneous utility as an index of 'sustainable development', which takes into account not only the welfare of the current generations but also the welfare of the future generations. In this context, the instantaneous utility can be thought of as a measure of the standard of living (Byrne (1997)) or comprehensively defined welfare that depends on both consumption and the environmental quality (Aghion and Howitt (1998)).

From (6.21) and (6.25), we see that consumption and pollution grow at the same rate, $g_c (= \frac{\delta - \rho}{\sigma})$, along the steady-state growth path, so the instantaneous utility function given in (6.1) becomes

$$U(t) = \frac{\left(c(0)e^{g_c t}\right)^{1-\sigma}}{1-\sigma} - \frac{\phi\left(x(0)e^{g_c t}\right)^{\gamma}}{\gamma} \tag{6.26}$$

in the steady state, where the initial consumption and pollution levels are given as $c(0) = c_0 > 0$ and $x(0) = x_0 > 0$, respectively. Differentiating the instantaneous utility function given in (6.26) with respect to time yields the change of the instantaneous utility along the steady-state growth path as

$$\dot{U}(t) = (c(0)^{1-\sigma} e^{g_c(1-\sigma)t} - \phi x(0)^{\gamma} e^{g_c \gamma t}) g_c . \tag{6.27}$$

Since $\sigma > 0$ and $\gamma > 1$, $g_c\gamma$ is greater than $g_c(1-\sigma)$, so the negative effect of pollution growth on utility dominates the positive effect of consumption growth along the steady-state growth path. Furthermore, the growth of the instantaneous utility, $\dot{U}(t)$, asymptotically approaches negative infinity, i.e.,

$$\lim_{t\to\infty}\dot{U}(t) = \lim_{t\to\infty} g_c e^{g_c\gamma t}\left(c(0)^{1-\sigma}e^{g_c(1-\sigma-\gamma)t} - \phi x(0)^{\gamma}\right) = -\infty, \tag{6.28}$$

which shows a sharp contrast with the social planner's problem in which the social utility improves along the asymptotic steady-state growth path.

In a decentralized economy without government intervention, the utility growth rate is

$$\frac{\dot{U}(t)}{U(t)} = \frac{\left(c(0)^{1-\sigma}e^{g_c(1-\sigma)t} - \phi x(0)^{\gamma}e^{g_c\gamma t}\right)g_c}{\left(\dfrac{c(0)^{1-\sigma}e^{g_c(1-\sigma)t}}{1-\sigma} - \dfrac{\phi x(0)e^{g_c\gamma t}}{\gamma}\right)} \tag{6.29}$$

along the steady-state growth path in which consumption and pollution grow at the same rate g_c. As we see from (6.29), the growth rate of utility is not constant over time. However, the asymptotic long-run growth rate of utility is given as

$$\lim_{t\to\infty}\frac{\dot{U}(t)}{U(t)}=\lim_{t\to\infty}\left(\frac{\gamma(1-\sigma)\left((c(0)e^{g_c t})^{1-\sigma}-\phi(x(0)e^{g_c t})^{\gamma}\right)}{\gamma(c(0)e^{g_c t})^{1-\sigma}-\phi(1-\sigma)(x(0)e^{g_c t})^{\gamma}}\right)g_c$$
$$=\lim_{t\to\infty}\left(\frac{\gamma(1-\sigma)\left(c(0)^{1-\sigma}e^{g_c(1-\sigma-\gamma)t}-\phi x(0)^{\gamma}\right)}{\gamma c(0)^{1-\sigma}e^{g_c(1-\sigma-\gamma)t}-\phi(1-\sigma)x(0)^{\gamma}}\right)g_c=\gamma g_c. \qquad (6.30)$$

Since $U(t) < 0$ and $\dot{U}(t) < 0$ as t goes to infinity, the instantaneous utility, a measure of the standard of living or comprehensively-defined welfare, decreases over time without bound at the asymptotic long-run growth rate of γg_c. In this context, sustainable development cannot be achieved in a decentralized economy without government intervention because the negative effect of pollution growth more than offsets the benefit of higher consumption, and consequently, consumers become progressively worse off.

Although our model does not take into account the critical threshold level[42] of environmental quality, below which economic growth or even life cannot be sustained, the results in this section give us an important lesson for economic growth and sustainable development. Unless the increased economic capacity is used for the environmental quality improvement as well as higher consumption, the only path to sustainable development might be the one of no economic growth because the environmental cost associated with the increased production is too high.

[42] Aghion and Howitt (1998) assume the existence of a critical ecological threshold by supposing that there is a finite lower limit of environmental quality below which cumulative environmental deterioration is irreversible, entailing a prohibitive cost. Also, Common and Perrings (1992) impose an ecological sustainability constraint on the allocation of economic resources in an economic growth model to ensure

6.3 Policy Analysis for Implementing the Social Optimum

In the decentralized economy without government intervention, we see that consumers become progressively worse off and sustainable development is not possible because pollution level increases without bound over time. By contrast, in the social planner's problem, we see that sustainable development can be achieved with the optimal control of pollution.

In this section, we study the issue of what policies might implement the optimal sustainable growth path, which is the third goal of our research in this chapter in order to examine the possibility of achieving sustainable development in a decentralized economy. We assume that the government has a policy tool to control pollution.

Specifically, two kinds of policy instruments are taken into account in our model: pollution tax and pollution voucher (permit). We examine whether any of these instruments might implement the social optimum in a decentralized economy as in the social planner's problem.

Let k_0, $h_0 > 0$ be given, and let $\{c^*(t), \omega^*(t), u^*(t), k^*(t), h^*(t), \lambda_1^*(t), \lambda_2^*(t), t \geq 0\}$ be the optimal paths for consumption and others, which are found by solving the social planner's problem. Also, let $x^*(t) \equiv \omega^*(t)^{\alpha(\beta+1)} k^*(t)^{\alpha} \left((1-u^*(t))h^*(t)\right)^{1-\alpha}$ be the optimal path of pollution.

6.3.1 Pollution Tax

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the stability of the global system that is composed of disturbed (exploited) and undisturbed resources.

Suppose the government wants to implement the social optimum by imposing a pollution tax, $\tau(t)$, to firms. We assume that the representative household and the representative firm are the same as before in Section 6.1.

6.3.1.1 The Household's Problem

The household owns physical and human capital, and one unit of non-leisure time. The household takes as given the rates of return on physical and human capital, r(t) and w(t), respectively. The household receives rental income, r(t)k(t), by renting physical capital to firms. Also, the household with human capital h(t) receives wage income, w(t)(1-u(t))h(t), by devoting the fraction, (1-u(t)), of non-leisure time to production. Although the pollution level, x(t), varies over time by the firm's decision under government regulation, the household takes the pollution level as given. We assume that government returns the pollution tax revenue, $R(t) = \tau(t)x(t)$, to the household as a lump sum subsidy. The lump sum subsidy from government, R(t), which is a part of household's income, is also taken as given by the household. Given the household's income, r(t)k(t) + w(t)(1-u(t))h(t) + R(t), the household makes decisions on consumption, saving (physical capital accumulation), and the allocation of non-leisure time between two sectors to maximize the lifetime utility.

So the representative household's problem is

$$\max \int_0^\infty e^{-\rho t}\left(\frac{c(t)^{1-\sigma}}{1-\sigma}-\frac{\phi x(t)^\gamma}{\gamma}\right)dt \tag{6.31}$$

s.t. $\dot{k}(t) = r(t)k(t) + w(t)(1 - u(t))h(t) + R(t) - c(t)$,

$\dot{h}(t) = \delta u(t)h(t)$,

where $k(0) = k_0 > 0$, $h(0) = h_0 > 0$ are given, and the household take as given r(t), w(t), R(t), and x(t).

The current value Hamiltonian for the representative household's problem is

$$H = \frac{c^{1-\sigma}}{1-\sigma} - \frac{\phi x^\gamma}{\gamma} + \lambda_1(rk + w(1-u)h + R - c) + \lambda_2 \delta uh, \tag{6.32}$$

where λ_1 and λ_2 denote the costate variables associated with physical and human capital, respectively.

Since the household's problem here is essentially the same as that in Section 6.1 except R(t), which the household takes as given, the first-order conditions with respect to c and u and the Euler equations for λ_1 and λ_2 are the same as those in Section 6.1. If we rewrite those conditions, they are as below:

$$\frac{\partial H}{\partial c} = 0 \Rightarrow c^{-\sigma} = \lambda_1, \tag{6.33}$$

$$\frac{\partial H}{\partial u} = 0 \Rightarrow \lambda_1 w = \lambda_2 \delta \text{ or } w = \frac{\lambda_2}{\lambda_1}\delta . \tag{6.34}$$

$$\dot{\lambda}_1 = \rho\lambda_1 - \frac{\partial H}{\partial k} \Rightarrow \frac{\dot{\lambda}_1}{\lambda_1} = \rho - r(t), \tag{6.35}$$

$$\frac{\dot{\lambda}_2}{\lambda_2} = \rho - \delta . \tag{6.36}$$

Comparing these conditions for a maximum in a decentralized economy with those for the social planner's problem, we see that the optimum can be implemented if and only if the paths for λ_1, λ_2, and hence for consumption are identical with the optimal ones. If we assume that all the variables except R(t) start at the same levels as those in the social planner's problem, then the growth rate and the time path of λ_2 are the same as those obtained from the social planner's problem. In the social planner's problem, the Euler equation for λ_1 is

$$\frac{\dot{\lambda}_1}{\lambda_1} = \begin{cases} \rho - \alpha\left(\frac{(1-u)h}{k}\right)^{1-\alpha} + \left(\frac{\alpha}{\beta+1}\right)\left(\frac{(1-u)h}{k}\right)^{1-\alpha}\left[\lambda_1\psi\left[k^{\alpha}((1-u)h)^{1-\alpha}\right]^{-\gamma}\right]^{-1}, & \text{if } \omega = 1, \\ \rho - \left(\frac{\alpha\beta}{\beta+1}\right)\left(\frac{(1-u)h}{k}\right)^{1-\alpha}\omega^{\alpha}, & \text{if } \omega < 1, \end{cases} \tag{6.37}$$

where all notations are the same as before; α and β are parameters of production technology and pollution function, respectively, $\psi = \frac{1}{\phi(\beta+1)}$, and $\eta = \frac{1}{\gamma(\beta+1)-1} > 0$. Also, ω is a quality index of differentiated physical capital that is used by firms, and the optimal value of ω in the social planner's problem is given as

$$\omega = \begin{cases} 1, & \text{if } \lambda_1 \geq \frac{1}{\psi}\left[k^{\alpha}((1-u)h)^{1-\alpha}\right]^{\gamma-1}, \\ \left[\psi\lambda_1\left[k^{\alpha}((1-u)h)^{1-\alpha}\right]^{1-\gamma}\right]^{\frac{\eta}{\alpha}}, & \text{if } \lambda_1 < \frac{1}{\psi}\left[k^{\alpha}((1-u)h)^{1-\alpha}\right]^{\gamma-1}. \end{cases} \tag{6.38}$$

Thus, the path for λ_1 in (6.35) matches with the optimal one if and only if the equilibrium rental rate on physical capital, $r^*(t)$, should satisfy

$$r^*(t) = \begin{cases} \alpha\left(\frac{h_y^*(t)}{k^*(t)}\right)^{1-\alpha}\left[1 - \frac{1}{(\beta+1)\psi\lambda_1^*(t)}\left(k^*(t)^{\alpha} h_y^*(t)^{1-\alpha}\right)^{\gamma-1}\right], & \text{if } \omega^*(t) = 1, \\ \frac{\alpha\beta}{\beta+1}\left(\frac{h_y^*(t)}{k^*(t)}\right)^{1-\alpha}\omega^*(t)^{\alpha}, & \text{if } \omega^*(t) < 1, \end{cases} \tag{6.39}$$

where $h_y^*(t) \equiv (1-u^*(t))h^*(t)$ and $\omega^*(t)$ is the optimal value given in (6.38). In this case, the equilibrium rate of return on human capital is determined as

$$w^*(t) = \delta \frac{\lambda_2^*(t)}{\lambda_1^*(t)}. \tag{6.40}$$

The fraction, $u^*(t)$, of non-leisure time devoted to human capital accumulation is automatically determined[43] if $h_y^*(t)$ and the growth rate of $h_y^*(t)$ (and $h^*(t)$) are determined by solving the firm's problem.

6.3.1.2 The Firm's Problem

Now we turn to the firm's problem to solve for the competitive equilibrium. The representative firm's problem is to maximize profit at each point of time t, taking as given the rental rates of physical and human capital, r(t), w(t), and the pollution tax, $\tau(t)$. As before, physical capital is differentiated into infinitely many physical capital goods in terms of productivity and pollution-generating level. Since government regulation through pollution tax restricts the use of lower quality but more productive physical capital, more physical capital is needed to produce the same amount of output with less pollution. Therefore, the firm's problem is a static one to choose its input levels, k and h_y, and the type of differentiated physical capital, ω, to maximize profit at any given point of time as below:

$$\max_{\omega,k,h_y} \pi = (\omega k)^\alpha h_y^{1-\alpha} - \left[rk + wh_y + \tau\left((\omega^\beta (\omega k))^\alpha h_y^{1-\alpha}\right)\right] \qquad (6.41)$$

s.t. $\quad \omega \le 1$.

In effect, the firm's choice of the type of differentiated physical capital is essentially the same as that of how much pollution to emit in the production process. Thus, for analytical convenience, it is equivalent to use the model in which pollution is treated as a normal input of production. In this case, the firm makes decisions on its input levels, k, h_y, and x[44] to maximize profit, taking r, w, and τ as given:

$$\max_{k,h_y,x} \pi = \left(k^{\alpha} h_y^{1-\alpha}\right)^{1-\frac{1}{\beta+1}} x^{\frac{1}{\beta+1}} - (rk + wh_y + \tau x) \tag{6.42}$$

$$\text{s.t.} \qquad x \le k^{\alpha} h_y^{1-\alpha}.$$

The Lagrangian for this problem is

$$L = \left(k^{\alpha} h_y^{1-\alpha}\right)^{1-\frac{1}{\beta+1}} x^{\frac{1}{\beta+1}} - (rk + wh_y + \tau x) + \mu(k^{\alpha} h_y^{1-\alpha} - x), \tag{6.43}$$

where $\mu \ge 0$ is the Lagrange multiplier associated with the inequality constraint for pollution input, x.

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[43] Note that $u(t) = \frac{1}{\delta}\frac{\dot{h}(t)}{h(t)}$.

[44] Here, we note that pollution is an input of production rather than a byproduct. The production function in our basic model can be transformed into the one with pollution as a normal input of production. Appendix D derives the production function with pollution as a normal input.

Solving the Kuhn-Tucker condition for a maximum subject to inequality constraint yields that

$$\frac{\partial L}{\partial \mu} = k^{\alpha} h_y^{1-\alpha} - x \geq 0,\ \mu \geq 0, \text{and if } x < k^{\alpha} h_y^{1-\alpha}, \text{then } \mu = 0. \qquad (6.44)$$

Thus, the first-order conditions for a maximum with respect to x, k, and h_y are

$$\frac{\partial L}{\partial x} = 0 \Rightarrow \tau \leq \frac{1}{\beta+1}\left(\frac{x}{k^{\alpha} h_y^{1-\alpha}}\right)^{\frac{1}{\beta+1}-1}, \qquad (6.45)$$

$$\frac{\partial L}{\partial k} = 0 \Rightarrow r \geq \alpha\left(\frac{\beta}{\beta+1}\right)\frac{(k^{\alpha} h_y^{1-\alpha})^{1-\frac{1}{\beta+1}} x^{\frac{1}{\beta+1}}}{k}, \qquad (6.46)$$

$$\frac{\partial L}{\partial h_y} = 0 \Rightarrow w \geq (1-\alpha)\left(\frac{\beta}{\beta+1}\right)\frac{(k^{\alpha} h_y^{1-\alpha})^{1-\frac{1}{\beta+1}} x^{\frac{1}{\beta+1}}}{h_y}, \qquad (6.47)$$

with equality if $x < k^{\alpha} h_y^{1-\alpha}$. Since the production technology exhibits constant returns to scale to its inputs, k, h_y, and x, the firm's zero-profit condition is satisfied as below:

$$\left(k^{\alpha} h_y^{1-\alpha}\right)^{1-\frac{1}{\beta+1}} x^{\frac{1}{\beta+1}} = rk + wh_y + \tau x. \qquad (6.48)$$

If the inequality constraint is not binding (i.e., $x < k^{\alpha} h_y^{1-\alpha}$), then the first-order conditions for profit maximization are enough to determine the equilibrium prices. In order to implement the social optimum, government should set the tax rate so that pollution level is the same as that obtained from the social planner's problem:

$$x = (\psi\lambda_1)^{(\beta+1)\eta} (k^{\alpha} h_y^{1-\alpha})^{\beta\eta}, \quad \text{if} \quad x < k^{\alpha} h_y^{1-\alpha}. \tag{6.49}$$

Thus, when the inequality constraint is not binding, in the sense that pollution is optimally controlled, the equilibrium prices that implement the social optimum are obtained by substituting (6.49) into (6.45), (6.46), and (6.47):

$$\tau = \frac{1}{\beta+1} \left(\psi\lambda_1 (k^{\alpha} h_y^{1-\alpha})^{1-\gamma} \right)^{-\beta\eta}, \tag{6.50}$$

$$r = \alpha \left(\frac{\beta}{\beta+1} \right) \left(\frac{h_y}{k} \right)^{1-\alpha} \left(\psi\lambda_1 (k^{\alpha} h_y^{1-\alpha})^{1-\gamma} \right)^{\eta}, \tag{6.51}$$

$$w = (1-\alpha) \left(\frac{\beta}{\beta+1} \right) \left(\frac{k}{h_y} \right)^{\alpha} \left(\psi\lambda_1 (k^{\alpha} h_y^{1-\alpha})^{1-\gamma} \right)^{\eta}. \tag{6.52}$$

The competitive prices given in (6.50), (6.51), and (6.52) can also be obtained by solving the firm's problem given in (6.41) and substituting the optimal value of ω,

$\omega = \left(\psi\lambda_1 (k^{\alpha} h_y^{1-\alpha})^{1-\gamma}\right)^{\frac{\eta}{\alpha}}$, into the first-order conditions to implement the social optimum.

On the other hand, if the inequality constraint is binding, in the sense that pollution is not controlled at all, then the highest tax is $\frac{1}{\beta+1}$, below which the firm will use the dirtiest technology, so the equilibrium tax rate can lie anywhere in the interval

$$\tau \in \left[0, \frac{1}{\beta+1}\right]. \tag{6.53}$$

In this case, both the first-order conditions and zero-profit condition should be used to determine the equilibrium rates of return on physical and human capital as below:

$$r \geq \alpha\left(\frac{\beta}{\beta+1}\right)\left(\frac{h_y}{k}\right)^{1-\alpha}, \quad w \geq (1-\alpha)\left(\frac{\beta}{\beta+1}\right)\left(\frac{k}{h_y}\right)^{\alpha},$$
$$rk + wh_y = (1-\tau\varphi^{1-\alpha})k^{\alpha}h_y^{1-\alpha}, \text{ and } \frac{rk}{wh_y} = \frac{\alpha}{1-\alpha}. \tag{6.54}$$

Combining (6.53) and (6.54), we can solve for the equilibrium prices of physical and human capital, in terms of the equilibrium tax rate, and their possible ranges:

$$r = \alpha(1-\tau)\left(\frac{h_y}{k}\right)^{1-\alpha} \Rightarrow r \in \left[\alpha\left(\frac{\beta}{\beta+1}\right)\left(\frac{h_y}{k}\right)^{1-\alpha}, \alpha\left(\frac{h_y}{k}\right)^{1-\alpha}\right], \tag{6.55}$$

$$w = (1-\alpha)(1-\tau)\left(\frac{k}{h_y}\right)^{\alpha} \Rightarrow r \in \left[(1-\alpha)\left(\frac{\beta}{\beta+1}\right)\left(\frac{k}{h_y}\right)^{\alpha}, (1-\alpha)\left(\frac{k}{h_y}\right)^{\alpha}\right]. \tag{6.56}$$

Hence at the input prices given by (6.50), (6.51), and (6.52), (or (6.53), (6.55), and (6.56)), the firm will choose its input levels, x, k, and h_y, so that the pollution level is the same as the optimal one obtained from the social planner's problem.

6.3.1.3 Equilibrium

Now we put the household and firm's problems together to solve for the competitive equilibrium under the government tax policy for pollution control.

First, when the inequality constraint is not binding, i.e., $x(t) < k(t)^{\alpha} h_y(t)^{1-\alpha}$, or $\omega(t) < 1$, the rental rate on physical capital, r(t), obtained by solving the firm's problem given in (6.51) exactly matches with the rental rate on physical capital given in (6.39). Therefore, it satisfies the condition for the equilibrium path of λ_t given in (6.35), which is obtained by solving the household's problem, to be the same as the optimal one. Hence, the equilibrium tax rate and return on human capital, which implement the social optimum for periods when $x(t) < k(t)^{\alpha} h_y(t)^{1-\alpha}$, are determined by (6.50) and (6.52), respectively.

Second, when the inequality constraint is binding so that $x(t) = k(t)^{\alpha} h_y(t)^{1-\alpha}$, or $\omega(t) = 1$, the equilibrium values of τ, r, and w should lie in the intervals given in (6.53), (6.55), and (6.56), respectively. Since the optimal rental rate on physical capital given in (6.39) lies in the interval of (6.55), the equilibrium tax rate that implements the optimum is determined by combining the first equation in (6.39) and equation in (6.55). Also, the equilibrium tax rate solves the equilibrium prices of physical and human capital given in (6.55) and (6.56), and consequently, the competitive equilibrium prices of x, k, and h_y are given as

$$\tau^*(t) = \begin{cases} \dfrac{\left(k^*(t)^{\alpha} h_y^*(t)^{1-\alpha}\right)^{\gamma-1}}{(\beta+1)\psi\lambda_1^*(t)}, & \text{if } x^*(t) = k^*(t)^{\alpha} h_y^*(t)^{1-\alpha} \\ \dfrac{\left(\psi\lambda_1^*(t)\left(k^*(t)^{\alpha} h_y^*(t)^{1-\alpha}\right)^{1-\gamma}\right)^{-\beta\eta}}{\beta+1}, & \text{if } x^*(t) < k^*(t)^{\alpha} h_y^*(t)^{1-\alpha}, \end{cases} \tag{6.57a}$$

or alternatively,

$$\tau^*(t) = \begin{cases} \dfrac{\left(k^*(t)^{\alpha} h_y^*(t)^{1-\alpha}\right)^{\gamma-1}}{(\beta+1)\psi\lambda_1^*(t)}, & \text{if } \omega^*(t) = 1, \\ \dfrac{\omega^*(t)^{-\alpha\beta}}{\beta+1}, & \text{if } \omega^*(t) < 1, \end{cases} \tag{6.57b}$$

$$r^*(t) = \alpha(1-\tau^*(t))\left(\frac{h_y^*(t)}{k^*(t)}\right)^{1-\alpha}, \tag{6.58}$$

$$w^*(t) = (1-\alpha)(1-\tau^*(t))\left(\frac{k^*(t)}{h_y^*(t)}\right)^{\alpha}. \tag{6.59}$$

From equations (6.57a) and (6.57b), it is interesting to note that the optimal tax rate is not zero, but strictly positive, even when the firms do not make efforts to reduce pollution by using the dirtiest technology. To put it in another way, firms pay the positive taxes for pollution even when the tax has no effect on pollution reduction. However, the positive tax rate in effect reduces the market rates of return on physical and human capital. The lower rates of return reduce the incentive for the new investment in physical and human capital, and hence, preventing pollution from increasing at a faster rate over time.

At the prices $\{r^*(t), w^*(t), \tau^*(t)\}$ given as the above in (6.57), (6.58), and (6.59), the firm will choose the optimal inputs $\{k^*(t), h_y^*(t), x^*(t)\}$ at any point in time t. The resulting path for the quality of differentiated physical capital used by firms, $\omega^*(t)$, which can be viewed as an index of emission standard, is also optimal. Since government tax revenue, $R^*(t) = \tau^*(t)\, x^*(t)$, is distributed to households as a lump sum subsidy, the household's income is equal to the sum of returns to physical and human capital, and a lump sum subsidy from government. Given $\{r^*, w^*, x^*, R^*\}$, the values $\{c^*, k^*, h_y^*, u^*, h^*, \lambda_1^*, \lambda_2^*\}$ that solve the household's problem are the same as those obtained from the social planner's problem, so the competitive equilibrium is a social optimum. Thus, the social optimum can be implemented in a decentralized economy at all times with a pollution tax, $\tau^*(t)$.

Finally, consider the long-run growth rate of the pollution tax. Taking logarithms and differentiating the second equation in (6.57b) yields the long-run growth rate of pollution tax as below:

$$\frac{\dot{\tau}}{\tau} = -\alpha\beta g_\omega = \frac{\gamma - 1 + \sigma}{\gamma} g_y > 0, \tag{6.60}$$

where $g_\omega < 0$ and $g_y > 0$ are the long-run growth rates of ω and y, respectively. Along the steady-state growth path when $\omega(t) < 1$, the pollution tax rate should rise at a constant rate as above. It implies that as economic growth increase the value of environmental quality, the firms have to pay a higher price for pollution input. Thus, the firms are willing to substitute more conventional inputs (physical and human capital) for pollution, which induces the firms to choose cleaner technology, leading the improvement in environmental quality as in the social planner's problem.

6.3.2 Pollution Voucher

Suppose the government distributes pollution vouchers to each firm by the same amount at each time t, and allows a secondary market for the vouchers. The households and firms are the same as before. We assume that the firm's profit, if there is any, is distributed to households. In order to implement the social optimum, the quantity of pollution vouchers that government distributes to each firm is $x^*(t)$, which is the optimal level of pollution obtained by solving the social planner's problem. With pollution vouchers, the firms are entitled to emit pollution by the amount of $x^*(t)$. Otherwise, firms can buy (sell) pollution vouchers in a secondary market for pollution vouchers to pollute more (less) than $x^*(t)$.

The representative firm's problem is to maximize profit, taking input prices as given, at each point in time. The firm makes decisions about how much physical and human capital to employ, and how much pollution to generate (or equivalently, which type of differentiated physical capital to use) in production. Therefore, the representative firm's problem is the static one as below:

$$\max_{k,h_y,x} \pi = \left(k^\alpha h_y^{1-\alpha}\right)^{1-\frac{1}{\beta+1}} x^{\frac{1}{\beta+1}} - \left(rk + wh_y + p_x(x - x^*)\right) \tag{6.61}$$

$$\text{s.t.} \qquad x \le k^\alpha h_y^{1-\alpha},$$

where p_x represents the price of a pollution voucher, and x^* is the quantity of pollution vouchers that each firm receives from government. Since the firm takes x^* as given, and consequently, x^* does not affect the firm's decisions, the firm's problem under pollution voucher scheme, given in (6.61), is basically the same as that under pollution tax, given in (6.42), if we replace p_x by τ. The first-order and break-even conditions, evaluated at the market clearing quantity, $x = x^*$, determine the equilibrium prices of inputs. The only difference from pollution tax is that the firm's profit is equal to the market value of pollution vouchers received from government, $p_x x^*$.

Next, consider the household's problem. Since the firm's profit, $\pi(t) = p_x(t)x^*(t)$, is distributed to households, each household's income consists of the firm's profit in addition to the rental and wage income earned by supplying physical and human capital to firms. Given the household's income, the household makes decisions about consumption, physical and human capital accumulation to maximize the lifetime utility. Thus, the representative household's problem is

$$\max \int_0^\infty e^{-\rho t}\left(\frac{c(t)^{1-\sigma}}{1-\sigma} - \frac{\phi x(t)^\gamma}{\gamma}\right)dt \tag{6.62}$$

$$\text{s.t.} \quad \dot{k}(t) = r(t)k(t) + w(t)(1-u(t))h(t) + \pi(t) - c(t),$$

$$\dot{h}(t) = \delta u(t)h(t).$$

The household's problem given in (6.62) is the same as that with pollution tax, given in (6.31), if we replace $\pi(t)$ by $R(t)$. Since $\pi(t)$ or $R(t)$ does not affect the household's decisions, the first-order conditions are exactly the same as those under pollution tax. As before, the social optimum is implemented with pollution vouchers if and only if the rental rate of physical capital satisfies (6.39).

Combining the firm's and household's problems, we see that the competitive equilibrium price of a pollution voucher is essentially the same as the pollution tax rate, given in (6.57a) and (6.57b), to implement the social optimum, i.e.,

$$p_x^*(t) = \begin{cases} \dfrac{\left(k^*(t)^\alpha h_y^*(t)^{1-\alpha}\right)^{\gamma-1}}{(\beta+1)\psi\lambda_1^*(t)}, & \text{if } x^*(t) = k^*(t)^\alpha h_y^*(t)^{1-\alpha} \\ \dfrac{\left(\psi\lambda_1^*(t)\left(k^*(t)^\alpha h_y^*(t)^{1-\alpha}\right)^{1-\gamma}\right)^{-\beta\eta}}{\beta+1}, & \text{if } x^*(t) < k^*(t)^\alpha h_y^*(t)^{1-\alpha}, \end{cases} \tag{6.63a}$$

or alternatively,

$$p_x^*(t) = \begin{cases} \dfrac{\left(k^*(t)^\alpha h_y^*(t)^{1-\alpha}\right)^{\gamma-1}}{(\beta+1)\psi\lambda_1^*(t)}, & \text{if } \omega^*(t) = 1, \\ \dfrac{\omega^*(t)^{-\alpha\beta}}{\beta+1}, & \text{if } \omega^*(t) < 1, \end{cases} \tag{6.63b}$$

where x^* and ω^* are the optimal pollution level and the optimal quality of differentiated physical capital, respectively. Therefore, with the competitive equilibrium prices of inputs, k, h_y, and x, the resulting paths for consumption, physical and human capital, and pollution are the same as the optimal ones obtained from the social planner's problem. Furthermore, if we assume that the pollution vouchers are distributed to households rather than firms, the household receives income from the sale of pollution vouchers, $\pi(t)$, as the household receives a lump sum subsidy, R(t), from government under tax policy. In this case, the household's and firm's problems are the same as those with pollution tax if we replace $\pi(t)$ by R(t), and the equilibrium solutions are exactly the same as those under pollution tax policy. Therefore, the pollution voucher scheme also implements the social optimum in a decentralized economy.

6.3.3 Direct Regulation of the Pollution Level

Now we assume that the government directly sets the emission standard on how much pollution is allowed to emit in production, say $x^*(t)$, at each time t. In contrast with the previous model of pollution tax or voucher, in which the optimal level of pollution coincides with the optimal quality of differentiated physical capital, the direct regulation mechanism yields quite different aspects in the firm's decision making behavior, depending on which the government regulates; i.e., the pollution level or the type of differentiated physical capital used in production. Intuitively, under direct regulation of the pollution level, the firm faces no cost at all for the allowed level of pollution, but only a benefit from producing more output, so the firm will emit pollution by the maximum level at each time t, $x^*(t)$, set by the government. Furthermore, since the government does not impose any restriction on the type of differentiated physical capital used in production, the firm can choose any type of physical capital as input. In this case, the firm will not choose to use the cleaner physical capital at the cost of less output produced, but to use the dirtiest physical capital (i.e., $\omega(t) = 1$), in order to maximize its profit. Thus, the firm's problem is to choose its input levels of k and h_y, taking the pollution level, $x^*(t)$, as given, to maximize its profit.

In order to take into account the firm's choice of the type of differentiated physical capital, we write the firm's problem as

$$\max_{\omega,k,h_y} \pi = (\omega k)^{\alpha} h_y^{1-\alpha} - rk - wh_y \tag{6.64}$$

$$\text{s.t.} \quad x^* \geq \omega^{\alpha(\beta+1)} k^{\alpha} h_y^{1-\alpha},$$

$$\omega \leq 1.$$

Since $\frac{\partial \pi}{\partial \omega} > 0$, it is obvious that the firm's profit is maximized when $\omega = 1$ and $x^* = \omega^{\alpha(\beta+1)} k^{\alpha} h_y^{1-\alpha}$, i.e., the firm will choose to use the dirtiest physical capital in production and emit pollution by the level of x^*. In this context, government's restriction on the type of differentiated physical capital used in production seems to be a more stringent environmental regulation than limiting the pollution level for the resource allocation for pollution abatement. Direct regulation of the pollution level is not useful for the government to induce firms to use the cleaner technology in production. Solving the first-order conditions will yield the equilibrium rental rates of physical and human capital as

$$r = \alpha \left(\frac{h_y}{k} \right)^{1-\alpha}, \tag{6.65}$$

$$w = (1-\alpha)\left(\frac{k}{h_y}\right)^{\alpha}. \tag{6.66}$$

The equilibrium rental rates of physical and human capital under direct regulation of the pollution level are clearly different from the optimal rental rates of physical and human capital, (6.58) and (6.59), determined by pollution tax. Thus, the household, whose behavior depends on the rates of return on physical and human capital determined by (6.65) and (6.66), cannot choose the optimal paths for consumption and investments in physical and human capital. Consequently, the social optimum cannot be implemented by the direct regulation of the pollution level either.

Based on the results that we have derived so far under different policy tools, we see that government should choose the pollution tax or voucher scheme rather than this type of direct regulation to implement the social optimum in a decentralized economy. The intuition behind these results is that the effectiveness of government policy depends on the market mechanism associated with pollution price.

Under the pollution tax or voucher system, it is implicitly assumed that price of pollution is determined in a separate market, and the firm buys the right to pollute at the given price of pollution. So, the market rental rates of physical and human capital reflect their true values, which enable the household to make the optimal decisions about saving (physical capital accumulation) and the fraction of non-leisure time devoted to human capital accumulation.

Under direct regulation of pollution, however, there is no market for pollution, and accordingly, firms are entitled to emit the allowed level of pollution at no cost. The market rental rates of physical and human capital include the price of pollution, i.e., the value of the right to pollute. In other words, the market rental rate of physical (human) capital accounts for not only the true return on physical (human) capital, but also the value of the right to emit additional units of pollution for each unit of physical (human) capital. Since the market returns on physical and human capital are higher than the true (optimal) returns, these overvalued market returns on physical and human capital are misleading the household's behavior into over investment in physical and human capital. As a consequence, the social optimum is not implemented under direct regulation. These results may help to provide the theoretical basis for the increasing use of the "incentive-based" mechanisms for environmental regulation rather than the "command-and control" approaches in the U.S. and around the world (Hahn (2000)).

Chapter 7

Summary and Conclusions

This dissertation undertakes an extensive analysis of the interaction between economic growth and the environment. We have developed a simple theoretical model that is consistent with the empirical evidence of an inverted U-shaped pattern of pollution relative to income, the so-called environmental Kuznets curve. This model, however, is different from the previous studies modeling the link between growth and environment. In particular, we present a unique and realistic specification of the pollution generating process, and we incorporate the environmental externality into an endogenous growth model of human capital with differentiated physical capital. Utilizing our analytical framework, we have studied different issues regarding economic growth and the environment, such as: (i) theoretical analysis of an environmental Kuznets curve, (ii) the long-run growth and sustainable development in the presence of pollution, and (iii) a policy analysis for implementing the social optimum in a decentralized economy.

In Chapter 4, we present a simple theoretical model that yields consistent results with the empirical evidence of an inverted U-shaped pattern of pollution relative to income in both static and dynamic settings. Although many empirical studies find the inverted U-shaped relationship between per capita income and pollution levels, the reason for such a relationship has been left open in that research. Hence, we make a contribution to the literature on growth and the environment by providing a theoretical basis to explain why pollution follows an inverted U-shaped pattern with respect to income level.

Both of our static and dynamic models show that one of the important factors determining the pattern of pollution growth with respect to income is the potential output level of the economy. We find that it is optimal not to control pollution when potential output is low but it may be optimal to control it at higher level of potential output. Since our result indicates that it is optimal not to control pollution when the potential output level is relatively low, it supports the position of less-developed countries that claim that they should be allowed to have a less stringent environmental policy for faster economic growth.

Another important factor that is crucial in determining the income-pollution relationship is the elasticity of marginal utility of consumption (σ_c) relative to the elasticity of substitution in production between pollution and conventional inputs (σ_y). A higher level of σ_c implies that consumers are willing to give up more consumption to reduce a given amount of pollution as income level increases. On the other hand, a higher level of σ_y implies that if pollution is treated as an input of production, firms are willing to use more conventional inputs, instead of reducing the pollution input, in response to an increase in the price of the pollution input. If both σ_c and σ_y are high enough for σ_c to be greater than $1/\sigma_y$, the price of pollution will increase sharply with income and firms will reduce pollution by a large amount in response to a higher price of pollution.

Therefore, if we assume that σ_c is greater than $1/\sigma_y$ (σ_y equals one for the Cobb-Douglas production technology), an inverted U-shaped relationship between per capita income and pollution is derived from both of our static and dynamic models. As the potential output level grows, the economy has a transition from the region in which pollution is not controlled to the region in which pollution is controlled. When pollution is not controlled at all, it is clear that pollution increases with income. However, when potential output exceeds a critical level, it is optimal to control pollution. Moreover, the pollution level decreases if and only if σ_c is greater than $1/\sigma_y$, so in this case the optimal behavior of pollution displays an inverted U-shaped pattern with respect to income level.

Since we believe that one of the important objectives for the study of economic growth is to explore its implications on welfare, we have used our model to address the issue of sustainable development in Chapter 5. We interpreted the term "sustainable development" as development that improves the quality of life, which depends on environmental quality as well as consumption of produced goods. In this context, we analyzed the long-run growth of the instantaneous utility, which depends on both consumption and pollution levels, as an index of sustainable development. Thus, one of the major contributions of this dissertation lies in applying the concept of sustainable development to a model of economic growth. For an extensive analysis of the long-run behaviors of the economy in the presence of pollution, we have studied both cases in which pollution has its impact on utility as a flow and as a stock.

When pollution affects utility as a flow, the economy has a transition from the initial growth path in which the dirtiest physical capital is used in production, to the asymptotic long-run growth path in which pollution is controlled by using cleaner types of physical capital in production. The turning point of the economy in transition depends on the potential output level relative to the shadow value of physical capital. After the economy's potential output exceeds the critical level of output, the economy asymptotically approaches the steady-state growth path along which physical capital, output, and consumption grow at a common, constant rate. However, human capital grows faster than physical capital along the asymptotic long-run growth path in the presence of pollution, while the long-run growth rate of human capital is the same as that of physical capital if there is no environmental consideration. Along the asymptotic long-run growth path, the pollution level increases but more slowly than output if $0 < \sigma_c < 1$, and declines if $\sigma_c > 1$, where σ_c represents the elasticity of marginal utility of consumption. When pollution is linked with output production, the social planner chooses the cleaner production technology by allocating resources to internalize the negative externality. Optimal control of pollution leads to quality improvement in physical capital used in production, while human capital plays an important role as a source of sustained growth of the economy. We have shown that long-run growth and sustainable development are achieved with more stringent environmental policy, provided that the social marginal product of human capital is not affected by the presence of pollution.

When the disutility of pollution is caused by the cumulative stock of pollutants, the long-run growth implications are almost the same as those derived from the model with pollution flow. The growth rates along the asymptotic long-run growth path for this model are the same as those obtained from the previous model in which pollution affects utility as a flow. The dynamic behavior of the pollution stock displays the same pattern as that of the pollution flow in the previous model, and the asymptotic long-run growth rate of pollution is also the same as before, and is independent of the natural decay rate. However, the path of pollution during transition will generally be different from that of the previous model with pollution flow, because the decisions for pollution control, in terms of both timing and strictness, in this model depend on the shadow value of physical capital relative to that of pollution stock. Therefore, depending on how we treat pollution, the peak level of pollution may occur at different levels of income. In contrast with the pollution flow model, it was shown that the natural decay rate of pollution should not be too small for the existence of optimal solution when pollution has its impact as a cumulated stock.

In Chapter 6, we have studied the equilibrium growth paths and the possibility of sustainable development in a decentralized economy without government intervention. We have also dealt with the issue of implementing the social optimum in a decentralized economy by introducing different instruments of government policy.

Our results indicate that consumption, output, physicalcapital, and human capital all grow at the same rate when there is no government intervention in a decentralized economy. We have found that the long-run growth rates of consumption, output, and physical capital are higher in a decentralized economy with no government intervention than those in the social planner's problem, and that the reverse is true for human capital. Pollution does not display the inverted U-shaped pattern with respect to growth. Furthermore, in contrast with the social planner's problem in which pollution growth depends on the elasticity of the marginal utility of consumption, the pollution level is unambiguously increasing at the same rate as that of output in a decentralized economy.

Moreover, it was shown that consumers become progressively worse off, even though the long-run growth rate of consumption in a decentralized economy is greater than that in the social planner's problem. Because the negative effect of the continued growth of pollution on utility more than offsets the benefit of higher consumption, the instantaneous utility decreases over time without bound, and consequently, it turned out that sustainable development, as an index of welfare improvement, cannot be achieved without government intervention

As for the implementation problem, we have shown that both the pollution tax and voucher schemes can implement the social optimum. The intuition behind these results is that the effectiveness of government policy depends on the market mechanism associated with the price of pollution and the rates of return on physical and human capital. If pollution is regulated using a pollution tax or voucher, then the pollution input has its own market price that is separate from the rental rates of physical and human capital. Therefore, the market rental rates of physical and human capital reflect their true (i.e., socially optimal) values under the pollution tax and voucher systems, which enable the household to make optimal decisions for investment in physical and human capital.

On the contrary, if we assume the government directly regulates the pollution level, then producers can emit the allowed level of pollution without paying for the right to emit the corresponding level of pollution. In this case, the rental rates of physical and human capital are not true (i.e., socially optimal) ones, but reflect the sum of their true returns and the price of pollution. Since the overvalued returns on physical and human capital mislead the household's behavior into over-investment in physical and human capital, this type of direct regulation cannot implement the social optimum in a decentralized economy. Thus, we conclude that the social optimum and sustainable development cannot be achieved without government intervention in a decentralized economy, and that a pollution tax or voucher scheme should be chosen to implement the social optimum rather than direct regulation of the pollution level.

The results that we have discussed so far suggest some important issues ignored in this dissertation for further research. The basic framework of the model we have developed in this dissertation can be extended to study a number of related issues.

Since pollution can be defined in a broad sense as the extractive use of the natural environment, and the environment can be modeled as a renewable resource because of the nature of regenerative capacity (e.g., López (1994), Bovenberg and Smulders (1995), and Aghion and Howitt (1998)), our model of pollution (stock) can be used to deal with the issue of renewable natural resource such as the optimal management of forests or fisheries. However, many resources are not renewable but exhaustible on a finite planet (e.g., fossil fuel, metals). Recent studies incorporating ecological concerns into the endogenous growth framework have been focused almost exclusively on analyzing the pollution problem. Because one of the objectives of this paper is to address the issue of sustainable development, we can also take into account the problem of nonrenewable natural resources as a factor of production to determine whether or not it limits growth in an endogenous growth framework.

In this dissertation, we assumed that there are infinitely many differentiated types of physical capital, which could be used for producing output. This implies that a cleaner production technology is always available. It would be interesting to study an economy in which we model endogenous pollution-abatement technological progress, which should be distinguished from the growth-enhancing technology improvement. The key issues that we could deal with in this framework include the question of whether there is sufficient market incentive to develop pollution-abatement technologies in the private sector. Also, we could discuss the role of government not only because of the market failure associated with environmental externality, but because the pollution-abatement technology has a public good character.

Our analysis of economic growth and the environment in this dissertation was based on a closed economy. With the emergence of trade liberalization, such as the North American Free Trade Agreement, there have been conflicting arguments about the interaction between trade and the environment. It would be interesting to consider an open economy to investigate how the pattern of trade, income levels, and the environment are affected by a free trade policy in developed vs. less-developed countries. In this case, for example, we must assume that different countries have different production technologies and different policies for environmental regulation.

Throughout this dissertation, population size was assumed to be an exogenously given constant. However, population growth is related to per capita income and pollution in the sense that an expansion of the aggregate labor force could raise the per capita growth rate if there is a scale effect in the economy and that pollution damage depends on the economy's population density. Therefore, population growth should be an important factor to be taken into account when investigating the linkage between economic growth and the environment. If the population growth rate is assumed to be positive, however, then our model of human capital would yield no balanced growth path, but the growth rate would explode because of the scale effect. In this context, we need to develop a new framework, such as non-scale model of economic growth (e.g., Eicher and Turnovsky (1999)), in order to study the effect of population growth on per capita income and pollution levels.

Finally, it would be worth calibrating our model to real data of both developed and less-developed countries to examine the pattern of pollution with respect to income at different stages of economic growth. Based on the calibration results, we might be able to suggest policy prescriptions on when and how strictly the pollution should be controlled in each country.

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Appendix A

Asymptotic Steady-State Growth Path in the Model of Pollution Flow

The socially optimal solution for the dynamic model in which pollution is treated as a flow is found by solving the social planner's problem given in (4.17). The current value Hamiltonian of this problem is given in (4.18) in the main text as

$$H = \frac{c^{1-\sigma}}{1-\sigma} - \frac{\phi}{\gamma}\left[\omega^{\alpha(\beta+1)}k^{\alpha}\left((1-u)h\right)^{1-\alpha}\right]^{\gamma} + \lambda_1\left[\omega^{\alpha}k^{\alpha}\left((1-u)h\right)^{1-\alpha} - c\right] + \lambda_2\delta uh + \mu(1-\omega), \text{or}$$

$$H = \frac{c^{1-\sigma}}{1-\sigma} - \frac{\phi x^{\gamma}}{\gamma} + \lambda_1(y-c) + \lambda_2\delta uh + \mu(1-\omega),$$

where λ_1 and λ_2 denote the costate variables associated with k and h, respectively, and μ is a Lagrange multiplier associated with the inequality constraint that the quality index, z, of the differentiated physical capital used in production should be greater than or equal to zero, and hence, $\omega \equiv \frac{1}{1+z} \leq 1$. The Kuhn-Tucker condition implies that the optimal values of μ and ω must meet the following condition: $\mu(1-\omega)=0$; therefore,

$$\omega < 1 \Rightarrow \mu = 0, \quad \mu \geq 0 \Rightarrow \omega = 1.$$

The first-order conditions with respect to c, u, and ω are

$$\frac{\partial H}{\partial c} = 0 \Rightarrow c^{-\sigma} = \lambda_1, \tag{A.1}$$

$$\frac{\partial H}{\partial u} = 0 \Rightarrow \lambda_2 \delta h = \frac{1-\alpha}{1-u}\left(\lambda_1 y - \phi x^{\gamma}\right), \tag{A.2}$$

$$\frac{\partial H}{\partial \omega} = 0 \Rightarrow -\phi\alpha(\beta+1)\frac{x^{\gamma}}{\omega} + \alpha\frac{\lambda_1 y}{\omega} - \mu = 0. \tag{A.3}$$

By applying the Kuhn-Tucker condition, we can rearrange equation (A.3) as

$$\text{If } \omega = 1 \text{, then } \lambda_1 y \geq \phi(\beta+1)x^{\gamma} \text{, or} \tag{A.4a}$$

$$\lambda_1 y = \phi(\beta+1)x^{\gamma} \text{, if } \omega < 1. \tag{A.4b}$$

If we substitute the production and pollution functions into (A.4a) and (A.4b), we can solve for the optimal value of ω as below:

$$\omega = \begin{cases} 1, & \text{if } \lambda_1 \geq \frac{1}{\psi}\left[k^{\alpha}((1-u)h)^{1-\alpha}\right]^{\gamma-1}, \quad \text{(A.5a)} \\ \left[\psi\lambda_1\left[k^{\alpha}((1-u)h)^{1-\alpha}\right]^{1-\gamma}\right]^{\frac{\eta}{\alpha}}, & \text{if } \lambda_1 < \frac{1}{\psi}\left[k^{\alpha}((1-u)h)^{1-\alpha}\right]^{\gamma-1}, \quad \text{(A.5b)} \end{cases}$$

where $\eta = \frac{1}{\gamma(\beta+1)-1} > 0$ and $\psi = \frac{1}{\phi(\beta+1)}$.

As we can see in equations (A.5a) and (A.5b), the optimal quality of differentiated physical capital is divided into the corner and interior solutions depending on whether or not the inequality constraint is binding. In other words, the optimal strategy for pollution control, represented by ω, depends on the level of potential output, defined as $y_p(t) = k(t)^{\alpha}((1-u(t))h(t))^{1-\alpha}$, relative to the shadow value of physical capital, $\lambda_1(t)$. Taking logarithms and differentiating (A.1) with respect to time gives

$$\frac{\dot{c}}{c} = -\frac{1}{\sigma}\frac{\dot{\lambda}_1}{\lambda_1}. \tag{A.6}$$

It is clear from (A.6) that the shadow value of physical capital falls if consumption rises in a growing economy. In this case, there is a critical point of time in the evolution of λ_1, defined as τ such that $\lambda_1(\tau) = \frac{1}{\psi}\left[k(\tau)^{\alpha}((1-u(\tau))h(\tau))^{1-\alpha}\right]^{\gamma-1}$, before which there is no pollution control at all, and after which pollution should be optimally controlled by using the cleaner physical capital in the final output production at the cost of fewer output produced. Therefore, equations (A.5a) and (A.5b) indicate that if the potential output level and consumption grow over time, the economy has a transition from the initial growth path in which $\omega = 1$, to the following growth path in which $\omega < 1$. Once the potential output exceeds the critical level, defined as $y_c(t) = (\psi\lambda_1(t))^{\frac{1}{\gamma-1}}$, the rate of change in the type of differentiated physical capital used in production, as an index of the strictness for pollution control, can be obtained by taking logarithms and differentiating the equation (A.5b) with respect to time, t:

$$\frac{\dot{\omega}}{\omega} = \frac{1}{\alpha(\gamma(\beta+1)-1)}\left[\frac{\dot{\lambda}_1}{\lambda_1} + (1-\gamma)\left(\alpha\frac{\dot{k}}{k} + (1-\alpha)\left(\frac{\dot{h}}{h} - \left(\frac{u}{1-u}\right)\frac{\dot{u}}{u}\right)\right)\right]. \tag{A.7}$$

If we assume that consumption and the stocks of physical and human capital all grow at constant rates, and u is constant along the asymptotic long-run growth path, then $\frac{\dot{\omega}}{\omega}$ must be negative because $\beta > 0$, $\gamma > 1$, and the sign of the terms in the brackets in (A.7) is negative. Since ω is inversely related to the quality of differentiated physical capital, z, in terms of cleanliness, the reduction of ω in a growing economy implies that the optimal pollution control should be increasingly stringent with economic growth.

Next, we are in a position to check the Euler equations for λ_1 and λ_2 for the optimal dynamic allocation of physical and human capital. The Euler equation for λ_1 is

$$\dot{\lambda}_1 = \rho\lambda_1 - \frac{\partial H}{\partial k} \Rightarrow \dot{\lambda}_1 = \rho\lambda_1 - \left[-\alpha\phi\frac{x^\gamma}{k} + \alpha\frac{\lambda_1 y}{k} \right]. \qquad \text{(A.8)}$$

The Euler equation for λ_2 is

$$\dot{\lambda}_2 = \rho\lambda_2 - \frac{\partial H}{\partial h} \Rightarrow \dot{\lambda}_2 = \rho\lambda_2 - \left[-\phi(1-\alpha)\frac{x^\gamma}{h} + (1-\alpha)\frac{\lambda_1 y}{h} + \lambda_2\delta u \right]. \qquad \text{(A.9)}$$

By using (A.2), equation (A.9) can be simplified as

$$\frac{\dot{\lambda}_2}{\lambda_2} = \rho - \delta. \qquad \text{(A.10)}$$

In addition to the first-order conditions, we are also given the laws of motion of k and h, the final output production function, the pollution function, and the transversality conditions in order to derive the optimal solutions. The law of motion of k is

$$\dot{k} = y - c \Rightarrow \frac{\dot{k}}{k} = \frac{y}{k} - \frac{c}{k}. \qquad \text{(A.11)}$$

The law of motion of h is

$$\dot{h} = \delta uh \Rightarrow \frac{\dot{h}}{h} = \delta u . \tag{A.12}$$

The production function of final output is given as

$$y = \omega^{\alpha} k^{\alpha} \left((1-u)h\right)^{1-\alpha} . \tag{A.13}$$

Hence, the growth rate of final output is given by

$$\frac{\dot{y}}{y} = \alpha\frac{\dot{\omega}}{\omega} + \alpha\frac{\dot{k}}{k} + (1-\alpha)\left[\frac{\dot{h}}{h} - \left(\frac{u}{1-u}\right)\frac{\dot{u}}{u}\right]. \tag{A.14}$$

Pollution is generated by the following function:

$$x = \omega^{\alpha(\beta+1)} k^{\alpha} \left((1-u)h\right)^{1-\alpha} = \omega^{\alpha\beta} y \tag{A.15}$$

Taking logarithms and differentiating (A.15) with respect to time yields

$$\frac{\dot{x}}{x} = \alpha(\beta+1)\frac{\dot{\omega}}{\omega} + \alpha\frac{\dot{k}}{k} + (1-\alpha)\left[\frac{\dot{h}}{h} - \left(\frac{u}{1-u}\right)\frac{\dot{u}}{u}\right] = \alpha\beta\frac{\dot{\omega}}{\omega} + \frac{\dot{y}}{y}. \tag{A.16}$$

From (A.14) and (A.16), it is clear that the pollution level increases at the same rate as that of output when $\omega = 1$. Substitute (A.7) and (A.14) into the equation (A.16), then the growth rate of pollution in the region where $\omega < 1$ is given as

$$\frac{\dot{x}}{x} = (\beta + 1)\eta\frac{\dot{\lambda}_1}{\lambda_1} + \beta\eta\left[\alpha\frac{\dot{k}}{k} + (1-\alpha)\left(\frac{\dot{h}}{h} - \left(\frac{u}{1-u}\right)\frac{\dot{u}}{u}\right)\right], \tag{A.17}$$

where $\eta = \dfrac{1}{\gamma(\beta+1)-1} > 0$.

From now on, we focus on analyzing the dynamic behavior of the economy in the region where $\omega < 1$ in order to investigate the long-run growth implications. Assume that the economy asymptotically approaches the steady-state growth path after the economy's potential output exceeds the critical level. We define the steady-state growth path as a path along which all the optimality conditions are satisfied and all the variables such as c, y, k, h, ω, and x grow at constant (not necessarily the same and possibly zero) rates, while the allocation of non-leisure time between two sectors, u (or 1-u), is constant.

Substituting (A.4b) into (A.2) gives

$$\lambda_2\delta h = \frac{(1-\alpha)\beta}{(\beta+1)}\frac{\lambda_1 y}{(1-u)}. \tag{A.18}$$

Using equations (A.4b), (A.6), and (A.18), we can rewrite equation (A.8) as

$$\frac{\dot{\lambda}_1}{\lambda_1}=\rho-\frac{\alpha\beta}{\beta+1}\frac{y}{k}\Rightarrow\frac{\dot{c}}{c}=\frac{1}{\sigma}\left(\frac{\alpha\beta}{\beta+1}\frac{y}{k}-\rho\right). \tag{A.19}$$

Let g_ℓ denote the long-run (steady-state) growth rate of the interested variable ℓ, where $\ell = y, c, k$, and so on. If consumption grows at a constant rate along the steady-state growth path in the long run, then equation (A.19) implies that $\frac{y}{k}$ should be constant along the steady-state growth path. Also, if physical capital grows at a constant rate in the long run, then from (A.11), $\frac{c}{k}$ should remain constant along the steady-state growth path. As a consequence, consumption, physical capital, and output grow at a common constant rate in the long run; i.e.,

$$\frac{\dot{k}}{k}=\frac{\dot{y}}{y}=\frac{\dot{c}}{c}\Rightarrow g_k=g_y=g_c. \tag{A.20}$$

We assume that u is constant along the steady-state growth path. Taking logarithms and differentiating equation (A.18) with respect to time, and combining with equations (A.6), (A.10), (A.12), and (A.20), we arrive at

$$\dot{g}_h = (1-\sigma)g_y + \delta - \rho \text{, or alternatively, } 1 - u = \frac{1}{\delta}\left(\rho - (1-\sigma)g_y\right). \qquad (A.21)$$

Taking logarithms and differentiating equation (A.4b) with respect to time and substituting equations (A.6), (A.16), and (A.20) gives

$$g_\omega = \left(\frac{1-\gamma-\sigma}{\alpha\beta\gamma}\right)g_y. \qquad (A.22)$$

Since $\frac{y}{k}$ and u are constant along the asymptotic long-run growth path, taking logarithms and differentiating $\frac{y}{k}$ with respect to time should result in zero, so using (A.13) and (A.20), we have

$$\alpha g_\omega + (1-\alpha)g_h - (1-\alpha)g_y = 0. \qquad (A.23)$$

Now we have three unknowns g_y, g_h, and g_ω, and three equations (A.21), (A.22), and (A.23). Hence, solving equations (A.21), (A.22), and (A.23) simultaneously, and using (A.16) (or (A.17)) and (A.20), we get the long-run growth rates of y, c, k, h, and x as

$$g_y = g_k = g_c = \frac{1}{\sigma + \vartheta}(\delta - \rho) \text{ where } \vartheta = \frac{\sigma + \gamma - 1}{(1-\alpha)\beta\gamma} > 0, \tag{A.24}$$

$$g_h = (1 + \vartheta) g_y > g_y , \tag{A.25}$$

$$g_x = \frac{1-\sigma}{\gamma} g_y . \tag{A.26}$$

Also, using (A.12) or (A.21), we have the optimal long-run value of the fraction of non-leisure time devoted to human capital accumulation, u^*, as

$$u^* = \frac{1}{\delta}\left((1-\sigma)g_y - \rho\right) + 1 = \frac{1}{\delta}(\delta - \rho)\left(1 + (1-\sigma)(\sigma + \vartheta)^{-1}\right) = \frac{1}{\delta} g_h > 0 . \tag{A.27}$$

We must assume that δ is greater than ρ for the growth rate of consumption and the optimal value of the fraction of non-leisure time devoted to human capital accumulation, u, to be positive. In this case, we see that the long-run growth rate of ω is negative from (A.22), which implies that pollution is more and more strictly controlled with economic growth. As we stated earlier, pollution level increases at the same rate as that of output in the early stage of economic growth when $\omega = 1$. However, equation (A.26) indicates that pollution level decreases along the asymptotic long-run growth path if and only if σ is greater than one. Therefore, if we assume that $\sigma > 1$, the pollution displays an inverted U-shaped pattern over time.

Finally, two transversality conditions are

$$\lim_{t\to\infty} e^{-\rho t}\lambda_1(t)k(t) = 0 \Rightarrow \lim_{t\to\infty} e^{((1-\sigma)g_k-\rho)t}c(0)k(0) = 0\,, \tag{A.28}$$

$$\lim_{t\to\infty} e^{-\rho t}\lambda_2(t)h(t) = 0 \Rightarrow \lim_{t\to\infty} e^{-\rho t}\frac{1}{\delta}\left(\frac{1-\alpha}{1-u}\right)\left(\frac{\beta}{\beta+1}\right)\lambda_1(t)y(t) = 0\,. \tag{A.29}$$

Thus, the above transversality conditions hold if $(1-\sigma)g_k < \rho$, which is satisfied if either

$$\begin{cases} \sigma \geq 1, \text{ or} \\ 0 < \sigma < 1 \text{ and } g_k < \dfrac{\rho}{1-\sigma}\,. \end{cases} \tag{A.30}$$

Appendix B

Stability of the Optimal Steady-State Growth Path

In this appendix, we check the stability of the optimal steady-state growth path that is found in Appendix A. As before, we focus on the asymptotic steady-state growth path in which $\omega < 1$. Using the optimality conditions derived in Appendix A, we define κ, χ, and υ as

$$\kappa \equiv \frac{y}{k} = \left[\psi\lambda_1\left[k^{\alpha}((1-u)h)^{1-\alpha}\right]^{\gamma\beta}\right]^{\eta} k^{-1}, \tag{B.1}$$

$$\chi \equiv \frac{c}{k} = k^{-1}\lambda_1^{-\frac{1}{\sigma}}, \tag{B.2}$$

$$\upsilon \equiv 1 - u = \frac{(1-\alpha)\beta}{\delta(\beta+1)}\frac{\lambda_1 y}{\lambda_2 h}, \tag{B.3}$$

where $\eta = \dfrac{1}{\gamma(\beta+1)-1} > 0$ and $\psi = \dfrac{1}{\phi(\beta+1)}$.

Since physical capital, output, and consumption grow at a common, constant rate along the asymptotic long-run growth path, κ, χ and υ are constant in the long run. Hence, using the above conditions and the laws of motion for λ_2 and h, given in (A.10) and (A.12), respectively, we see that along the asymptotic steady-state growth path, the growth rates should satisfy

$$\eta\left(g_{\lambda_1} + \gamma\beta(\alpha g_k + (1-\alpha)g_h)\right) - g_k = 0, \tag{B.4}$$

$$g_{\lambda_1} = -\sigma g_k, \tag{B.5}$$

$$g_h = g_{\lambda_1} - (\rho - \delta) + g_k, \tag{B.6}$$

where $g_h = \delta u$. Therefore, from these conditions (B.4), (B.5), and (B.6), we can obtain the same long-run growth rates of k and h as those given in (A.24) and (A.25) in Appendix A. Now we can check the stability of the optimal steady-state growth path by taking log-linearization of κ, χ, and υ around the steady-state values in the long run. The laws of motion for k and λ_1 given in Appendix A, and the growth rate of υ in the region of $\omega < 1$ can be expressed in terms of κ, χ, and υ as below:

$$\frac{\dot{k}}{k} = \kappa - \chi \Rightarrow \chi^{*} = \kappa^{*} - g_k, \tag{B.7}$$

$$\frac{\dot{\lambda}_1}{\lambda_1} = \rho - \frac{\alpha\beta}{\beta+1}\kappa \Rightarrow \kappa^* = \frac{\beta+1}{\alpha\beta}(\rho - g_{\lambda_1}), \tag{B.8}$$

$$\frac{\dot{\upsilon}}{\upsilon} = -\frac{\alpha\beta}{\beta+1}\kappa + g_k + \delta\upsilon \Rightarrow \upsilon^* = \frac{\alpha\beta}{\delta(\beta+1)}\kappa^* - \frac{1}{\delta}g_k, \tag{B.9}$$

where κ^*, χ^*, and υ^* are steady-state values in the long run.

Differentiating κ, χ, and υ, given in (B.1), (B.2), and (B.3), with respect to time, substituting (B.7), (B.8), and (B.9), and let

$$\Theta \equiv \ln\left(\frac{\kappa}{\kappa^*}\right), \; \Lambda \equiv \ln\left(\frac{\chi}{\chi^*}\right), \text{ and } \Gamma \equiv \ln\left(\frac{\upsilon}{\upsilon^*}\right),$$

in order to take log-linearization of κ, χ, and υ around the asymptotic steady-state growth path. Then we see that

$$\begin{pmatrix} \dot{\Theta} \\ \dot{\Lambda} \\ \dot{\Gamma} \end{pmatrix} = \begin{pmatrix} -\left(\eta(\gamma-1) + \eta(1+(1-\alpha)\beta\gamma)\left(\frac{\alpha\beta}{\beta+1}\right)\right)\kappa^* & \eta(\gamma-1)\chi^* & 0 \\ \left(\frac{\alpha\beta}{\sigma(\beta+1)} - 1\right)\kappa^* & \chi^* & 0 \\ \left(1 - \frac{\alpha\beta}{\beta+1}\right)\kappa^* & -\chi^* & \delta\upsilon^* \end{pmatrix} \begin{pmatrix} \Theta \\ \Lambda \\ \Gamma \end{pmatrix} \equiv (A)\begin{pmatrix} \Theta \\ \Lambda \\ \Gamma \end{pmatrix}. \tag{B.10}$$

Using the transversality conditions given in (A.28) and (A.29) and the steady-state values of κ, χ, and υ given in (B.7), (B.8), and (B.9), we see that

$$\det(A) = -\eta\delta\frac{\alpha\beta}{\beta+1}\left(1+(1-\alpha)\beta\gamma+\frac{\gamma-1}{\sigma}\right)\kappa^{*}\chi^{*}\upsilon^{*} < 0, \tag{B.11}$$

$$\mathrm{tr}(A) = \eta\left(\frac{\gamma(\beta+1)}{\alpha}+\gamma(1+\alpha\beta)-2\right)(\rho+\sigma g_{k})-2g_{k}\,. \tag{B.12}$$

Therefore, the asymptotic steady-state optimal growth path is at least locally stable because det(A) < 0, even though the sign of tr(A) is indeterminate.

Appendix C

Asymptotic Steady-State Growth Path in the Model of Pollution Stock

The socially optimal solution for the model in which pollution affects utility as a stock is obtained by solving the social planner's problem given in (5.14) in the main text. The current value Hamiltonian of this problem is given as

$$H = \frac{c^{1-\sigma}}{1-\sigma} - \frac{\phi X^{\gamma}}{\gamma} + \lambda_1\left[\omega^{\alpha}k^{\alpha}((1-u)h)^{1-\alpha} - c\right] + \lambda_2 \delta uh - \lambda_3\left[\omega^{\alpha(\beta+1)}k^{\alpha}((1-u)h)^{1-\alpha} - \varepsilon X\right]$$
$$+ \mu(1-\omega), \text{ or}$$

$$H = \frac{c^{1-\sigma}}{1-\sigma} - \frac{\phi X^{\gamma}}{\gamma} + \lambda_1(y-c) + \lambda_2 \delta uh - \lambda_3(\omega^{\alpha\beta}y - \varepsilon X) + \mu(1-\omega),$$

where λ_1, λ_2, and λ_3 denote the costate variables associated with k, h, and X, respectively. Since the shadow value of pollution stock represents the marginal damage caused by a unit increase in pollution stock, the sign of the costate variable for X is reversed, so that $\lambda_3 > 0$. As before, μ is a Lagrange multiplier associated with the inequality constraint that the quality index, z, of the differentiated physical capital used in production is greater than or equal to zero, and therefore, $\omega \equiv \frac{1}{1+z} \leq 1$. Hence the Kuhn-Tucker condition is the same as before when pollution enters utility as a flow:

$$\frac{\partial H'}{\partial \omega} \leq 0,\ \omega \leq 1, \text{ and } \mu(1-\omega) = 0;$$

i.e., $\omega < 1 \Rightarrow \mu = 0,\ \mu \geq 0 \Rightarrow \omega = 1.$

The problem becomes more complicated than that in which pollution is treated as a flow, because we have now three state variables (k, h, and X) as well as three choice variables (c, u, and ω). The first-order conditions with respect to c and u are

$$\frac{\partial H}{\partial c} = 0 \Rightarrow c^{-\sigma} = \lambda_1, \tag{C.1}$$

$$\frac{\partial H}{\partial u} = 0 \Rightarrow \lambda_2 \delta h = \frac{1-\alpha}{1-u}\left(\lambda_1 y - \omega^{\alpha\beta}\lambda_3 y\right), \tag{C.2}$$

Combining with the Kuhn-Tucker condition, the first-order condition with respect to ω is

$$\begin{aligned} &\frac{\partial H}{\partial \omega} = \alpha\frac{\lambda_1 y}{\omega} - \alpha(\beta+1)\omega^{\alpha\beta}\frac{\lambda_3 y}{\omega} - \mu \leq 0,\ \omega \leq 1, \text{ and } \mu(1-\omega) = 0, \\ &\text{or } \frac{\partial H}{\partial \omega} = 0 \text{ if } \omega < 1 \Rightarrow \alpha\frac{\lambda_1 y}{\omega} - \alpha(\beta+1)\omega^{\alpha\beta}\frac{\lambda_3 y}{\omega} = 0. \end{aligned} \tag{C.3}$$

Taking logarithms and differentiating (C.1) with respect to time gives

$$\frac{\dot{c}}{c} = -\frac{1}{\sigma}\frac{\dot{\lambda}_1}{\lambda_1}. \tag{C.4}$$

By applying the Kuhn-Tucker condition, we can rearrange equation (C.3):

$$\text{If } \omega = 1 \text{, then } \lambda_1 \geq \lambda_3(\beta+1) \text{, or} \tag{C.5a}$$

$$\lambda_1 = \lambda_3(\beta+1)\omega^{\alpha\beta}, \text{ if } \omega < 1. \tag{C.5b}$$

Solving (C.5a) and (C.5b) for ω in terms of λ_1, λ_3, and other parameters gives

$$\omega = \begin{cases} 1, & \text{if } \lambda_1 \geq \lambda_3(\beta+1), \\ \left(\dfrac{\lambda_1}{\lambda_3(\beta+1)}\right)^{\frac{1}{\alpha\beta}}, & \text{if } \lambda_1 < \lambda_3(\beta+1). \end{cases} \tag{C.6}$$

If the stocks of physical capital and pollution are both small at the initial stage of economic growth, the shadow price of physical capital, λ_1, could be sufficiently high relative to that of the pollution stock, λ_3, for λ_1 to be greater than or equal to $\lambda_3(\beta+1)$. In this case, only the dirtiest physical capital is used in the final output production (i.e., $\omega = 1$). Thus, the gross inflow of new pollution generated in the process of final output production increases at the same rate as that of the final output, and both the inflow of new pollution and the pollution stock rise. Over time, however, the shadow value of physical capital (λ_1) falls, and the shadow value of pollution stock (λ_3) rises as the stocks of both physical capital and pollution grow. There is a critical point in time, defined by τ such that $\lambda_1(\tau) = \lambda_3(\tau)(\beta + 1)$, after which it becomes optimal to control pollution by using the higher quality (i.e., cleaner) physical capital in the final output production. Since the shadow value of physical capital relative to that of pollution stock, λ_1 / λ_3, declines monotonically,[45] ω declines accordingly by (C.6), which means that pollution is more strictly controlled as economy grows. As the economy asymptotically approaches the steady state after passing the transition path along which there exists a critical point, the quality of physical capital as an index of clean technology improves at the rate

$$\frac{\dot{\omega}}{\omega} = \frac{1}{\alpha\beta}\left(\frac{\dot{\lambda}_1}{\lambda_1} - \frac{\dot{\lambda}_3}{\lambda_3}\right) < 0. \qquad \text{(C.7)}$$

From now on we focus on the region where $\omega < 1$ in order to explore the long-run growth implication when pollution enters utility as a stock. The Euler equation for λ_1 is

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[45] It will be shown later that the growth rates of λ_1 and λ_3 (expressed in terms of the growth rate of output) along the steady-state growth path are $g_{\lambda_1} = -\sigma g_y$ and $g_{\lambda_3} = \frac{(\gamma-1)(1-\sigma)}{\gamma} g_y$, and that $g_y > 0$. Hence the growth rate of the ratio λ_1 / λ_3 is $g_{\lambda_1} - g_{\lambda_3} = -\left[\frac{\gamma-1}{\gamma} + \sigma\left(1 - \frac{\gamma-1}{\gamma}\right)\right] g_y < 0$; i.e., the ratio λ_1 / λ_3 declines monotonically along the steady-state growth path.

$$\dot{\lambda}_1 = \rho\lambda_1 - \frac{\partial H}{\partial k} \Rightarrow \dot{\lambda}_1 = \rho\lambda_1 - \frac{\alpha y}{k}(\lambda_1 - \lambda_3\omega^{\alpha\beta}). \tag{C.8}$$

Using equations (C.4) and (C.5b), the growth rate of consumption is given as

$$\frac{\dot{c}}{c} = \frac{1}{\sigma}\left[\left(\frac{\alpha\beta}{\beta+1}\right)\frac{y}{k} - \rho\right] \tag{C.9}$$

The Euler equation for λ_2 is

$$\dot{\lambda}_2 = \rho\lambda_2 - \frac{\partial H}{\partial h} \Rightarrow \dot{\lambda}_2 = \rho\lambda_2 - \left[(1-\alpha)\frac{\lambda_1 y}{h} + \lambda_2\delta u - (1-\alpha)\omega^{\alpha\beta}\frac{\lambda_3 y}{h}\right]. \tag{C.10}$$

Substituting (C.5b) into (C.2) gives

$$\lambda_2\delta h = \frac{\beta(1-\alpha)}{\beta+1}\frac{\lambda_1 y}{1-u}. \tag{C.11}$$

By using equations (C.11) and (C.5b), (C.10) can be simplified as

$$\frac{\dot{\lambda}_2}{\lambda_2} = \rho - \delta. \tag{C.12}$$

The Euler equation for λ_3 is

$$\dot{\lambda}_3 = \rho\lambda_3 + \frac{\partial H}{\partial X} \Rightarrow \dot{\lambda}_3 = (\rho + \varepsilon)\lambda_3 - \phi X^{\gamma-1}. \qquad \text{(C.13)}$$

As well as the first-order conditions derived as above, we should take into account the laws of motion of k, h, and X, the final output production function, and the transversality conditions for the optimal solutions. The law of motion of k is

$$\dot{k} = y - c \Rightarrow \frac{\dot{k}}{k} = \frac{y}{k} - \frac{c}{k}. \qquad \text{(C.14)}$$

As before, we are interested in investigating the long-run growth implications rather than the transition. Thus, we focus on the steady-state growth path along which all the variables grow at constant, although not necessarily the same, rates while the fraction of non-leisure time allocated to human capital accumulation is constant. Let g_ℓ denote the long-run (steady-state) growth rate of the interested variable ℓ, where ℓ = y, c, k, and so on. If consumption grows at a constant rate in the long run, then from (C.9), $\frac{y}{k}$ should be constant along the steady-state growth path. Likewise, if physical capital grows at a constant rate, then from (C.14), $\frac{c}{k}$ should remain constant in the steady state, which implies that consumption, physical capital, and output grow at a common constant rate in the long run; i.e.,

$$\frac{\dot{k}}{k} = \frac{\dot{y}}{y} = \frac{\dot{c}}{c} \Rightarrow g_k = g_y = g_c . \tag{C.15}$$

The law of motion of h is

$$\dot{h} = \delta uh \Rightarrow \frac{\dot{h}}{h} = \delta u . \tag{C.16}$$

The law of motion of X is

$$\dot{X} = \omega^{\alpha(\beta+1)} k^{\alpha} \left((1-u)h\right)^{1-\alpha} - \varepsilon X = \omega^{\alpha\beta} y - \varepsilon X. \tag{C.17}$$

The production function of final output is given as

$$y = \omega^{\alpha} k^{\alpha} \left((1-u)h\right)^{1-\alpha} . \tag{C.18}$$

Taking logarithms and differentiating (C.18) with respect to time, we get the relationship between the long-run growth rate of final output and those of other variables as

$$\frac{\dot{y}}{y} = \alpha\frac{\dot{\omega}}{\omega} + \alpha\frac{\dot{k}}{k} + (1-\alpha)\frac{\dot{h}}{h} \Rightarrow g_y = \alpha g_\omega + \alpha g_k + (1-\alpha) g_h . \tag{C.19}$$

Taking logarithms and differentiating equation (C.11) with respect to time and combining with equations (C.4), (C.12), (C.15), and (C.16), we get

$$g_h = (1-\sigma)g_y + \delta - \rho \text{, or alternatively, } 1-u = \frac{1}{\delta}\left(\rho - (1-\sigma)g_y\right). \tag{C.20}$$

Taking logarithms and differentiating equation (C.5b) with respect to time and using (C.9), we arrive at

$$-\sigma\frac{\dot{c}}{c} = \frac{\dot{\lambda}_3}{\lambda_3} + \alpha\beta\frac{\dot{\omega}}{\omega} \Rightarrow g_c = -\frac{1}{\sigma}(g_{\lambda_3} + \alpha\beta g_\omega) . \tag{C.21}$$

From equation (C.9), we see that $\frac{y}{k}$ is constant along the steady-state growth path. Substituting the production function given in (C.18) into $\frac{y}{k}$, taking logarithms and differentiating with respect to time, we have

$$\alpha g_\omega + (1-\alpha)g_h - (1-\alpha)g_k = 0 . \tag{C.22}$$

From the Euler equation for λ_3 given in (C.13), the growth rate of λ_3 can be derived as

$\frac{\dot{\lambda}_3}{\lambda_3} = \rho + \varepsilon - \phi \frac{X^{\gamma-1}}{\lambda_3}$. If λ_3 grows at a constant rate along the steady-state growth path,

then $\frac{X^{\gamma-1}}{\lambda_3}$ is also constant in the steady state, which implies

$$g_{\lambda_3} = (\gamma - 1) g_X . \tag{C.23}$$

Also, the law of motion of X given in (C.17) implies that $\frac{\dot{X}}{X} = \frac{\omega^{\alpha\beta} y}{X} - \varepsilon$. If the pollution stock grows (falls) at a constant rate along the steady-state growth path, $\frac{\omega^{\alpha\beta} y}{X}$ should remain constant along the long-run growth path, so that

$$\alpha\beta g_\omega + g_y - g_X = 0. \tag{C.24}$$

Now we can get the long-run growth rates, $g_y (= g_k = g_c)$, g_h, g_ω, g_X, and g_{λ_3} by solving equations (C.15), (C.18), (C.21), (C.22), (C.23), and (C.24), simultaneously. The asymptotic long-run growth rates of the interested variables that we get are

$$g_y = g_k = g_c = \frac{1}{\sigma + \vartheta}(\delta - \rho) \text{ where } \vartheta = \frac{\sigma + \gamma - 1}{(1-\alpha)\beta\gamma} > 0, \tag{C.25}$$

$$g_h = (1+\vartheta)g_y > g_y, \tag{C.26}$$

$$g_\omega = \left(\frac{1-\gamma-\sigma}{\alpha\beta\gamma}\right)g_y < 0, \tag{C.27}$$

$$g_X = \frac{1-\sigma}{\gamma}g_y, \tag{C.28}$$

$$g_{\lambda_3} = \frac{(\gamma-1)(1-\sigma)}{\gamma}g_y. \tag{C.29}$$

The asymptotic long-run growth rates for this model are exactly same as those for the previous model in which pollution enters utility as a flow. Furthermore, dynamic behavior of the pollution stock in this model displays the same pattern as that of pollution flow that affects utility in the previous model. If $\sigma > 1$ holds, the pollution stock increases with no pollution control in the early stage of economic growth, and decreases as the economy asymptotically approaches the steady-state growth path. Therefore, the pollution stock displays an inverted U-shaped pattern over time as long as $\sigma > 1$ holds. Also, using (C.16) or (C.18), we have the steady-state value of the allocation of the fraction of human capital as

$$u^{*} = \frac{1}{\delta}\left((1-\sigma)g_y - \rho\right) + 1 = \frac{1}{\delta}(\delta-\rho)\left(1+(1-\sigma)(\sigma+\vartheta)^{-1}\right) = \frac{1}{\delta}g_h > 0. \tag{C.30}$$

Finally, transversality conditions are

$$\lim_{t\to\infty} e^{-\rho t}\lambda_1(t)k(t) = 0 \Rightarrow \lim_{t\to\infty} e^{((1-\sigma)g_k-\rho)t}c(0)k(0) = 0, \tag{C.31}$$

$$\lim_{t\to\infty} e^{-\rho t}\lambda_2(t)h(t) = 0 \Rightarrow \lim_{t\to\infty} e^{-\rho t}\frac{1}{\delta}\left(\frac{1-\alpha}{1-u}\right)\left(\frac{\beta}{\beta+1}\right)\lambda_1(t)y(t) = 0. \tag{C.32}$$

$$\lim_{t\to\infty} e^{-\rho t}\lambda_3(t)X(t) = 0 \Rightarrow \lim_{t\to\infty} e^{-((1-\sigma)g_k-\rho)t}\lambda_3(0)X(0) = 0 \tag{C.33}$$

Thus, the above transversality conditions hold if $(1-\sigma)g_k < \rho$, which is satisfied if either $\sigma \geq 1$, or $0 < \sigma < 1$ and $g_k < \frac{\rho}{1-\sigma}$.

In this model, however, there is one thing to note about the existence of the optimal solution. From the law of motion of X given in (C.17), we see that

$$\frac{\omega^{\alpha\beta}y}{X} = g_X + \varepsilon = \frac{(1-\sigma)g_y}{\gamma} + \varepsilon > 0 \Rightarrow (1-\sigma)g_y > -\varepsilon\gamma. \tag{C.34}$$

Combining the transversality conditions with (C.34), we can conclude that the optimal solution of this model exists if and only if

$$-\varepsilon\gamma < (1-\sigma)g_y < \rho \ . \tag{C.35}$$

Hence, if σ is greater than one, the natural decay rate of pollution stock, ε, should not be too small for the existence of solution in this model. For example, if $\sigma > 1$ and $\varepsilon = 0$ (i.e., the environment does not have its own self-correcting nature for the environmental degradation), then the optimal solutions of this model may not exist.

Appendix D

Production Function with Pollution as a Normal Input

The production and pollution functions in our basic model are

$$y = \omega^{\alpha} k^{\alpha} h_y^{1-\alpha}, \tag{D.1}$$

$$x = \omega^{\alpha(\beta+1)} k^{\alpha} h_y^{1-\alpha}. \tag{D.2}$$

From (D.2), we get

$$\omega = \left(\frac{x}{k^{\alpha} h_y^{1-\alpha}} \right)^{\frac{1}{\alpha(\beta+1)}}. \tag{D.3}$$

By substituting equation (D.3) into (D.1), we can show that pollution enters production function as an input:

$$y = \left(k^{\alpha} h_y^{1-\alpha} \right)^{1-\frac{1}{\beta+1}} x^{\frac{1}{\beta+1}}. \tag{D.4}$$

Hence from equation (D.4), we find that the final output is in effect produced using physical capital, human capital, and pollution as inputs, with the Cobb-Douglas production technology.

However, unlike other inputs, there is an inequality restriction on the pollution input because there is an inequality constraint on ω in our basic model:

$$
\begin{aligned}
&\omega \leq 1 \\
&\Rightarrow \omega^{\alpha(\beta+1)} = \frac{x}{k^{\alpha} h_y^{1-\alpha}} \leq 1 \\
&\Rightarrow x \leq k^{\alpha} h_y^{1-\alpha}, \\
&\text{i.e., } x = \begin{cases} \omega^{\alpha(\beta+1)} k^{\alpha} h_y^{1-\alpha}, & \text{if } \omega < 1, \\ k^{\alpha} h_y^{1-\alpha}, & \text{if } \omega = 1. \end{cases}
\end{aligned}
\qquad (D.5)
$$

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